# II PUC – CHAPTER 03
# CURRENT ELECTRICITY

## Electric current

☞ Electric current = $\frac{\text{charge}}{\text{time}}$ i.e. $I = \frac{q}{t} = \frac{ne}{t}$

n – number of electrons flowing across the cross section of a conductor in time t.

☞ If a small charge dq flows in time dt then instantaneous current is $I = \frac{dq}{dt}$

☞ **Current due to circular motion of charge:**

Electric current $I = fq = \frac{q}{T} = \frac{qv}{2\pi r} = \frac{q\omega}{2\pi}$

f = frequency of revolution; v= velocity; r = radius of orbit; ω = angular velocity

For electron I = fe

## Current density

☞ Current density $J = \frac{I}{A} = \sigma E = \frac{E}{\rho}$ σ = electrical conductivity; ρ = resistivity

☞
- Current through a conductor is $I = \vec{J} \cdot \vec{A} = JA\cos\theta$
- $I = \int \vec{J} \cdot d\vec{A} = \int (J\cos\theta)dA$

## Drift velocity

☞ Velocity of random motion of electrons increases with increase in temperature, $v_r \propto \sqrt{T}$

☞ Relaxation time and drift velocity decreases with increase in temperature, $v_d \propto \frac{1}{\sqrt{T}}$

☞ Drift velocity of electrons $v_d = \frac{I}{nAe} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{e\tau E}{m} = \frac{Ve\tau}{\ell m}$

where n = electron density ($m^{-3}$), τ - relaxation time and m – mass of electron.

E – Electric field, V – p.d across the conductor, A – area of cross section, ℓ - length of the conductor. J – Current density; σ = electrical conductivity

☞ For a wire of non-uniform cross-section $v_d \propto \frac{1}{A} \propto \frac{1}{r^2}$

Current (I) through the wire is same whereas current density (J) changes

☞ Mobility of electrons $\mu = \frac{v_d}{E} = \frac{I}{nAeE} = \frac{I\ell}{nAeV} = \frac{e\tau}{m}$

## Effects of electric current

☞ Quantity of heat produced $H = I^2Rt = VIt = \frac{V^2t}{R}$ joules.

where I = current in the resistance R, t = time for which current flows

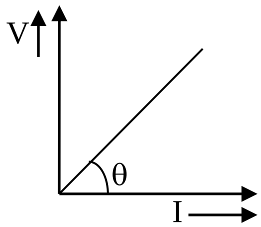
☞ If one electric heater boils a liquid in time $t_1$ and another heater boils the same liquid in time $t_2$, then time taken by the same liquid to boil when these two heater are

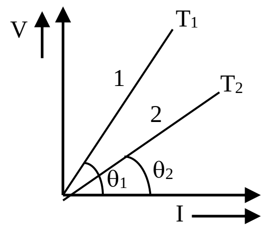
(i) in series is $t = t_1 + t_2$

(ii) in parallel is $t = \frac{t_1 t_2}{t_1 + t_2}$

**Ohm's Law**

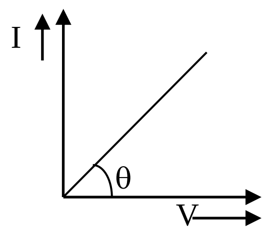
- $V \propto I$ or $V = IR$, R = electrical resistance of the conductor.
- $R = \frac{V}{I} = \frac{W}{QI}$
- Conductance $G = 1/R$ (Unit: siemen (S) or mho or $ohm^{-1}$ or A/V)
- For ohmic devices a plot of V vs I is a straight line
  Slope = $\tan\theta = V/I$ = resistance (R)




Here $\tan\theta_1 > \tan\theta_2$
So $R_1 > R_2$
i.e. $T_1 > T_2$

- A plot of I Vs V is a straight line
  Slope = $\tan\theta = I/V$ = conductance (G)




**Electric power**

- Electric power $P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$
- If bulbs of different powers are joined in parallel, then bulb with highest wattage glows brighter.
- If bulbs of different powers are joined in series, then bulb with lowest wattage glows brighter.
- Effective power of the combination when devices are in series $\frac{1}{P_s} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$
  when applied voltage is equal to the rated voltage.
- Effective power of the combination when devices are in parallel $P_P = P_1 + P_2 + P_3$
  when applied voltage is equal to the rated voltage.
- A wire fuses when square of the current (I) through it is directly proportional to the cube of its radius (r) i.e. $I^2 \propto r^3$

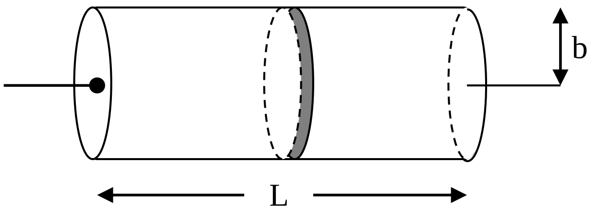
**Variation of resistance with its dimension:**

- Resistance of a conductor is directly proportional to its length (L) and inversely proportional to its area of cross section (A).
- Resistance $R = \frac{\rho L}{A}$ where ρ is called resistivity of the material of the conductor.
- Resistivity of the material of the conductor, $\rho = \frac{RA}{L}$
- Resistivity of a cylindrical conductor $\rho = \frac{\pi r^2 R}{L} = \frac{\pi d^2 R}{4L}$
  where R = resistance of conductor, A = area of cross-section,
  L = length of the conductor, d = diameter of the conductor
- Resistivity of the material $\rho = \frac{m}{ne^2\tau}$
  where m = mass of the electron, e = charge of the electron, n = number density of electrons, τ = relaxation time
- The reciprocal of resistivity is called conductivity $\sigma = 1/\rho$

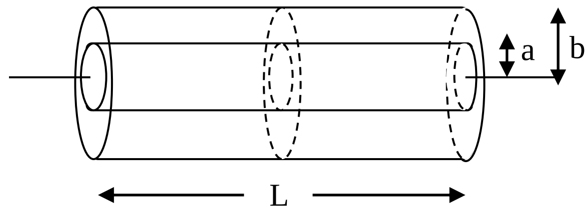
Also $\sigma = J/E$ where J – current density and E electric field between the ends of the conductor.

**Resistance of different shaped conductors**

☞ **Resistance of solid or hollow cylindrical wire**

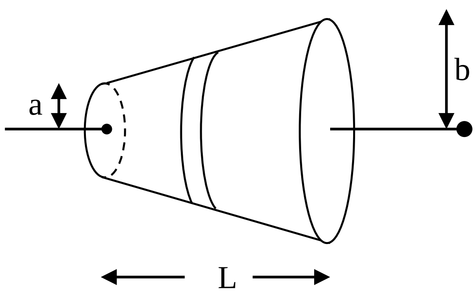



$$R = \frac{\rho L}{\pi b^2}$$




$$R = \frac{\rho L}{\pi (b^2 - a^2)}$$

☞ **Resistance of a wire of non-uniform area of cross section**




$$R = \frac{\rho L}{\pi ab}$$

(Here, $(b - a) << L$)

☞ **Resistance of stretched wire**

- When a wire is stretched $R_2 = R_1\left(\frac{\ell_2}{\ell_1}\right)^2 = R_1\left(\frac{A_1}{A_2}\right)^2 = R_1\left(\frac{r_1}{r_2}\right)^4$ i.e. $R \propto \ell^2 \propto \frac{1}{A^2} \propto \frac{1}{r^4}$

  $\ell_1$ and $\ell_2$ are the initial and final lengths.
  $A_1$ and $A_2$ are the initial and final area of cross section.
  $r_1$ and $r_2$ is the initial and final radius.
- If the length of metallic wire becomes n times, its resistance becomes $n^2$ times.
- If the radius of metallic wire becomes n times, its resistance becomes $\frac{1}{n^4}$ times.
- If the area of cross section of the metallic wire becomes n times, then its resistance becomes $1/n^2$ times.

**Variation of resistance of a conductor with temperature**

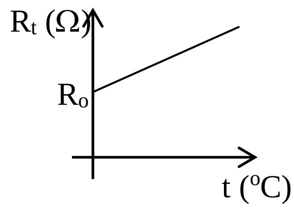
☞ The electrical resistance of most of the metallic conductors increases with increase in temperature according to $R_t = R_o (1 + \alpha t)$
$R_t$ = resistance of conductor at temperature $t^oC$ and $R_o$ = resistance of conductor at temperature $0^oC$
$\alpha$ is called the temperature coefficient of resistance of the material of the conductor.

☞ TCR of conductor is $\alpha = \frac{R_t - R_o}{R_o t} = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$

$R_1$ is resistance of conductor at $t_1\,^oC$, $R_2$ is resistance of conductor at $t_2\,^oC$

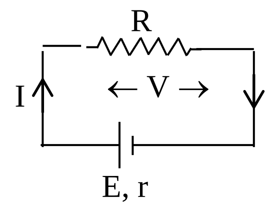
Also $\alpha \approx \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{\Delta R}{R_1 \Delta t}$

- A plot of $R_t$ Vs temperature of a conductor is a straight line whose slope is $R_o\alpha$ and y-intercept is $R_o$.
  Also $\alpha = \frac{\text{slope}}{\text{Y} - \text{int ercept}}$




- Variation of resistivity of a conductor with temperature $\rho_t = \rho_o (1 + \alpha t)$
  $\rho_t$ = Resistivity at $t^oC$, $\rho_o$ = Resistivity at $0^oC$.
  $\alpha$ = temperature coefficient of resistivity of the conductor.

**Simple circuit**




- Emf of the cell $E = I(R + r)$
- Current in the circuit $I = \frac{E}{R + r}$
  R = resistance of the external circuit
  r = internal resistance of the cell.
- Terminal P.d $V = IR = \frac{ER}{R + r} = E - Ir$
- P.d across the internal resistance $V' = Ir$
- Power dissipated in external resistance $P = VI = I^2R = \left(\frac{E}{R + r}\right)^2 R$
- Power dissipated is maximum when R = r
  $P_{max} = \frac{E^2}{4R}$
- Also V = E when $r \approx 0$ or $R = \infty$
  The emf of a cell is equal to the potential difference between the terminals of the cell, when the cell is in open circuit. i.e. when the current drawn from the cell is zero.
- When a cell is being charged, the current is given to the cell and the p.d across its terminal is always greater than its emf i.e. $V = E + Ir$ $(E < V)$

**Potentiometer**

- Potential gradient in a potentiometer $x = \frac{ER}{(R + r)L}$ (V/m)
  E – emf of the primary or standard cell, L - length of potentiometer wire,
  R – resistance of potentiometer wire, r – resistance of the standard cell.
- Internal resistance of a cell, $r = R\left(\frac{L - \ell}{\ell}\right)$ where L is the balancing length for the given cell in open circuit and $\ell$ is the balancing length for the same cell in closed circuit.
- **Comparisons of two cells:**
  Ratio of emfs of two cells $\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$
  $\ell_1$ and $\ell_2$ are balancing lengths for given cells.
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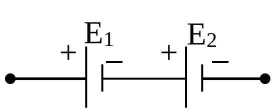
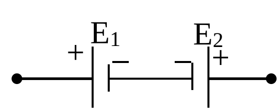


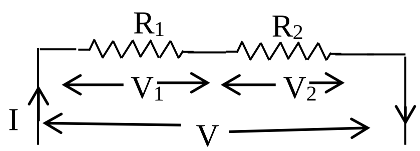

$(E_1 + E_2) = x\ell_1$ $(E_1 - E_2) = x\ell_2$

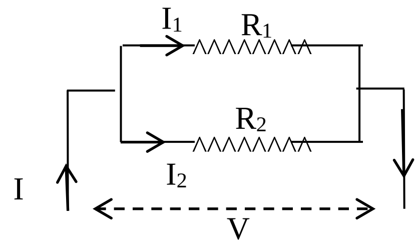
$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\ell_1}{\ell_2}$ or $\frac{E_1}{E_2} = \frac{\ell_1 + \ell_2}{\ell_1 - \ell_2}$

☞ Emf of the secondary cell = potential gradient × balancing length.

**Series or parallel combination of resistances**

1)



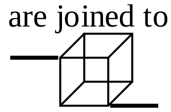




| Series | Parallel |
|---|---|
| Current through each resistance is same | P.d across each resistance is same |
| P.d across the combination $V = V_1 + V_2$<br>$V_1 = \frac{VR_1}{R_1 + R_2}$ and $V_2 = \frac{VR_2}{R_1 + R_2}$ | Main current $I = I_1 + I_2$<br>$I_1 = \frac{IR_2}{R_1 + R_2}$ and $I_2 = \frac{IR_1}{R_1 + R_2}$ |
| Effective resistance<br>$R_S = R_1 + R_2 + R_3 + \ldots + R_n$ | Effective resistance<br>$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$ |
| Effective resistance is always greater than highest value of resistance | Effective resistance is always less than least value of resistance |
| For two resistors $R_s = R_1 + R_2$ | For two resistors $R_P = \frac{R_1 R_2}{R_1 + R_2}$ |
| For n identical resistors $R_S = nR$<br>$V_1 = V_2 = V_3 \ldots = V/n$ | For n identical resistors $R_P = R/n$<br>$I_1 = I_2 = I_3 \ldots = I/n$ |
| Effective conductance<br>$\frac{1}{G_S} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3} + \ldots + \frac{1}{G_n}$ | Effective conductance<br>$G_P = G_1 + G_2 + G_3 + \ldots + G_n$ |
| P.d across each resistor $V \propto R$<br>$V_1:V_2:V_3 = R_1:R_2:R_3$ | Current through each resistor $I \propto \frac{1}{R}$<br>$I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$ |
| Power dissipated in each resistor $P \propto R$ | Power dissipated in each resistor $P \propto \frac{1}{R}$ |

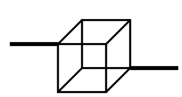
☞ Ratio of effective resistance of n identical resistors first connected in series and then in parallel $R_S/R_P = n^2$

☞ Twelve identical wires each of resistance R are joined to form a cube. Then equivalent resistance between

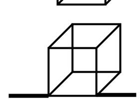
a) the ends of a face diagonal = $\frac{3R}{4}$



b) diagonally opposite corners = $\frac{5R}{6}$



c) any two adjacent corners = $\frac{7R}{12}$



**Branch currents**

☞ When two resistances are connected in parallel,

Branch current = $\frac{\text{Main current} \times \text{resistance of other branch}}{\text{sum of the resistance}}$

i.e. $I_1 = \frac{IR_2}{R_1 + R_2}$ or $I_2 = \frac{IR_1}{R_1 + R_2}$

☞ When resistances are connected in parallel branch current is also given by

$I_1 = \frac{IR_P}{R_1}$ or $I_2 = \frac{IR_P}{R_2}$ where $R_P$ is the effective resistance in parallel and I is the main current.

☞ When n identical resistances are connected in parallel then

Branch current = $\frac{\text{Main Current}}{\text{Number of branches}}$

**Combination of cells**

1)

| Series | Parallel |
|---|---|
| Cells are said to be in series such that negative terminal of a cell is connected to the positive terminal of the other cell and so on. | Cells are said to be in parallel if all negative terminals of the cells are connected together to one terminal and all positive terminals are connected to the other terminal. |
| Current through each cells is same | P.d across each cells is same |
| $V = V_1 + V_2 + V_3.......$<br>$V_1 = E_1 - Ir_1$; $V_2 = E_2 - Ir_2$ | $I = I_1 + I_2 + I_3.......$<br>$I_1 = \frac{E_1 - V}{r_1}$; $I_2 = \frac{E_2 - V}{r_2}$ |
| Equivalent internal resistance<br>$r_{eq} = r_1 + r_2 + r_3 + \ldots$ | Equivalent internal resistance<br>$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots$ |
| Equivalent emf $E_{eq} = E_1 + E_2 + E_3 + \ldots$ | $\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \ldots$ |
| Current through the circuit $I = \frac{E_{eq}}{R + r_{eq}}$ | Current through the circuit $I = \frac{E_{eq}}{R + r_{eq}}$ |
| Terminal p.d $V = E_{eq} - I r_{eq}$ | Terminal p.d $V = E_{eq} - I r_{eq}$ |

2) Combination of identical cells

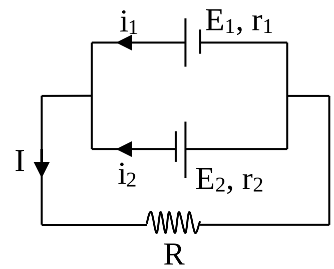
| For n identical cells in series | For m identical cells in parallel |
|---|---|
| Equivalent emf $E_{eq} = nE$ | Equivalent emf $E_{eq} = E$<br>(same as that of one cell) |
| Equivalent internal resistance $r_{eq} = nr$ | Equivalent internal resistance $r_{eq} = r/m$ |
| Current through the circuit $I = \frac{nE}{R + nr}$ | Current through the circuit is $I = \frac{mE}{mR + r}$ |
| Terminal p.d $V = E_{eq} - I r_{eq}$ | Terminal p.d $V = E_{eq} - I r_{eq}$ |
| P.d across each cell V' = V/n | Current through each cell I' = I/m |
| Current is maximum if R>>nr | Current is maximum if r >> mR |
| $I_{max} = \frac{nE}{R}$ | $I_{max} = \frac{mE}{r}$ |

- In series combination of n identical cells if N cells are oppositely connected then net emf of the circuit is E' = (n – 2N)E where E – Emf of each cell.

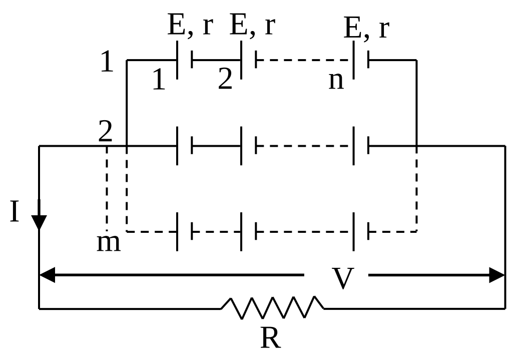
☞ If two cells of emfs $E_1$ and $E_2$ and internal resistances $r_1$ and $r_2$ are connected in parallel to an external resistance R

- Effective internal resistance $r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$
- Effective emf $E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$
- Current through external resistance, $I = \frac{E_{eq}}{R + r_{eq}}$
- If cells are connected with opposite polarity

  Effective emf $E_{eq} = \frac{E_1 r_2 \sim E_2 r_1}{r_1 + r_2}$




☞ **Mixed grouping:**




n = number of cells in each row; m = number of rows
Total number of cells = mn

- Equivalent emf of the combination $E_{eq} = nE$
- Equivalent internal resistance $r_{eq} = \frac{nr}{m}$
- Main current $I = \frac{mnE}{mR + nr}$
- Current through the circuit is maximum if external resistance is equal to the total internal resistance of the cells. i.e. $R = \frac{nr}{m}$

  Maximum current is $I_{max} = \frac{mE}{2r} = \frac{nE}{2R} = \frac{mnE}{2\sqrt{mnrR}}$

**Wheatstone's network**

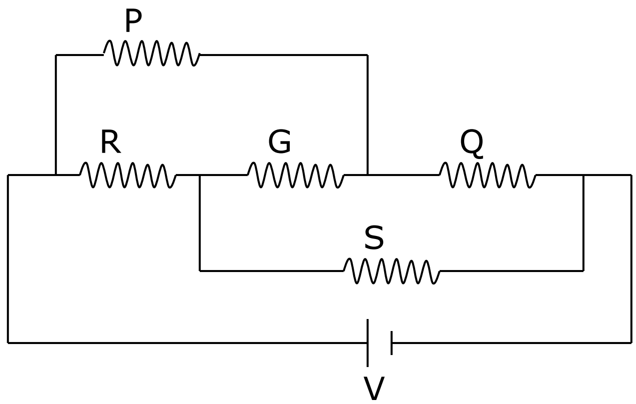
☞ The condition for electrical balance being $\frac{P}{Q} = \frac{R}{S}$

☞ Balanced condition is independent of galvanometer resistance, emf of the cell and its internal resistance.

☞ **In a balanced Wheatstone's network,**

- Current through the galvanometer $I_g = 0$
- P.d across galvanometer $V_g = 0$
- Ratio of adjacent resistances is equal. i.e. $\frac{P}{Q} = \frac{R}{S}$ or PS = QR
- Current through P & Q is equal. Also current through R and S are equal
- P.d across P = P.d across R and P.d across Q = p.d across S
- Effective external resistance of the circuit $R_{eff} = \frac{(P+Q)(R+S)}{P+Q+R+S}$
- When all the four resistances are identical then $R_{eff} = R$

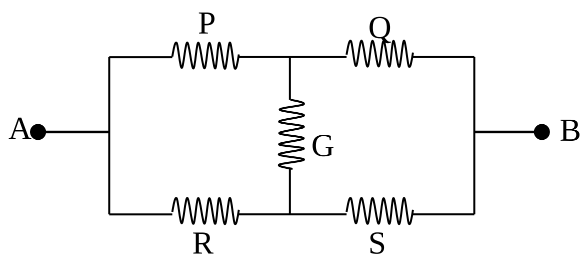
☞



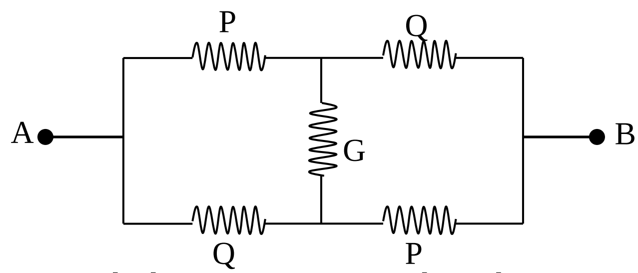

Above circuit is equivalent to Wheatstones network.

☞ **Equivalnet resistance of unbalanced network:**

$$R_{AB} = \frac{(P+Q)RS + PQ(R+S) + (P+Q)(R+S)G}{(P+Q+R+S)G + (P+R)(Q+S)}$$




$$R_{AB} = \frac{2PQ + (P+Q)G}{2G + P + Q}$$




**Metre Bridge**

☞ If P is the unknown resistance in the left gap, Q is the known resistance in the right gap. L is the balancing length (in m). Then P = $\frac{Q\,L}{1-L}$ Ω

☞ If L is in cm then P = $\frac{Q\,L}{100-L}$

☞ When P = Q, then balancing length L = 0.5 m or 50 cm.