II PUC - CHAPTER 02
ELECTRIC POTENTIAL AND CAPACITANCES

Electric potential

- Work done in moving a charge q_o from A to B is $W_{AB} = q_o(V_B - V_A)$
- **Relation between electric field and electric potential:**

If dV is the potential difference between two points separated by a distance dx, then

$$E = -\frac{dV}{dx} \text{ or } V = -\int \vec{E} \cdot \vec{dx}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\frac{dV}{dx}\hat{i} - \frac{dV}{dy}\hat{j} - \frac{dV}{dz}\hat{k}$$

P.d difference between B and A is $V_B - V_A = \frac{W_{AB}}{q_o} = -\int_B^A \vec{E} \cdot \vec{dr}$ where $\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

- Negative of the slope of the V-r graph denotes intensity of electric field i.e. $\tan\theta = -\frac{V}{r} = E$

Electric field and Electric potential:

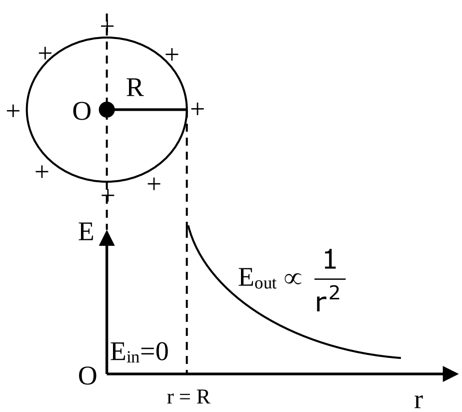
- **Point charge:**

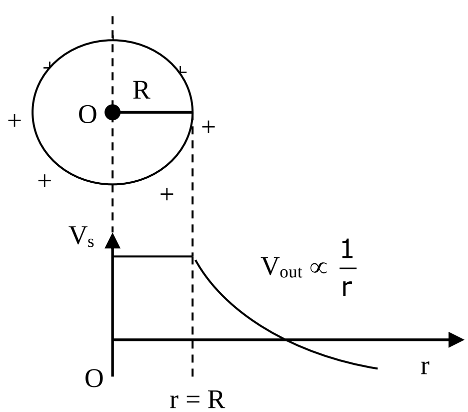
$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$ in free space or $\vec{E} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q}{r^2}\hat{r}$	$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} = Er$, in free space.
$E = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q}{r^2}$ in a medium r = distance of the point from the charge.	$V = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q}{r} = Er$, in a medium

- **Charged spherical shell of radius R:**

r = distance of the point from the centre of the sphere; $\sigma = \frac{q}{4\pi R^2}$ surface density of charge.

$E_{outside} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} = \frac{\sigma R^2}{\varepsilon_o r^2}$	$V_{outside} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} = \frac{\sigma R^2}{\varepsilon_o r} = E_{outside}r$
$E_{surface} = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^2} = \frac{\sigma}{\varepsilon_o}$	$V_{surface} = \frac{1}{4\pi\varepsilon_o}\frac{q}{R} = \frac{\sigma R}{\varepsilon_o} = E_{surface}R$
$E_{inside} = 0$	$V_{inside} = V_{surface}$; $P.d_{inside} = 0$





- For two charged spheres of same surface densities of charge, electric potential $V \propto R$

i.e. $\frac{V_1}{V_2} = \frac{R_1}{R_2}$ R_1 & R_2 are the radius of two spheres

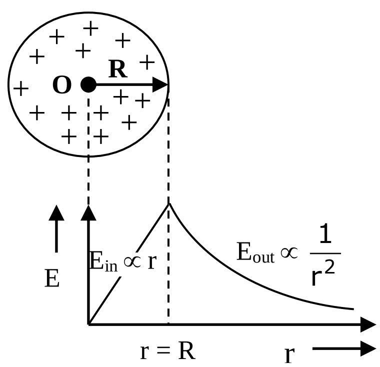
- When two charged spheres are placed in contact they share charges and attain same potential:

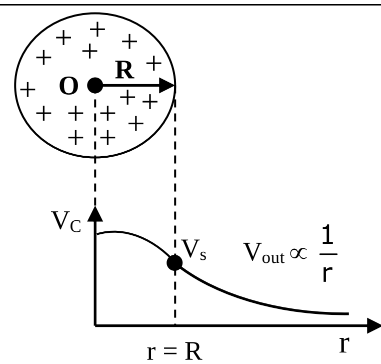
$q \propto R$; $E \propto R$, $\sigma \propto 1/R$

☞ **Uniformly charged solid non-conducting sphere of radius R:**

Volume density of charge $\rho = \frac{3q}{4\pi R^3}$ r = distance of the point from the centre

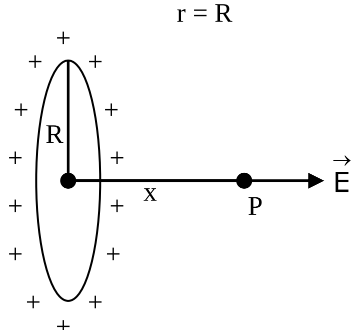
$E_{outside} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} = \frac{\rho R^3}{3\varepsilon_o r^2}$	$V_{outside} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} = \frac{\rho R^3}{3\varepsilon_o r}$
$E_{surface} = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^2} = \frac{\rho R}{3\varepsilon_o}$	$V_{surface} = \frac{1}{4\pi\varepsilon_o}\frac{q}{R} = \frac{\rho R^2}{3\varepsilon_o}$
$E_{inside} = \frac{1}{4\pi\varepsilon_o}\frac{q}{R^3}r = \frac{\rho r}{3\varepsilon_o}$	$V_{inside} = \frac{1}{4\pi\varepsilon_o}\frac{q(3R^2 - r^2)}{2R^3} = \frac{\rho(3R^2 - r^2)}{6\varepsilon_o}$
$E_{centre} = 0 \quad (\because r = 0)$	$V_{centre} = \frac{1}{4\pi\varepsilon_o}\frac{3q}{2R} = \frac{3}{2}V_{surface}$





☞ **Uniformly charged ring:**

x is the distance of the point from the centre of the ring. 'R' is the radius of the ring.



$E_{axis} = \frac{1}{4\pi\varepsilon_o}\frac{qx}{(R^2 + x^2)^{3/2}} = \frac{1}{4\pi\varepsilon_o}\frac{2\pi R\lambda x}{(R^2 + x^2)^{3/2}}$
At $x >> R$, $E = \frac{1}{4\pi\varepsilon_o}\frac{q}{x^2}$
At $x = 0$, $E_{centre} = 0$
If $x = \frac{R}{\sqrt{2}}$, $E_{max} = \frac{1}{6\sqrt{3}\pi\varepsilon_o}\frac{q}{R^2}$

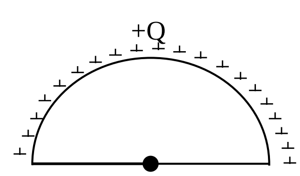
☞ **Uniformly charged segment of ring (charged arc)**

α = angle subtended by the arc at the point

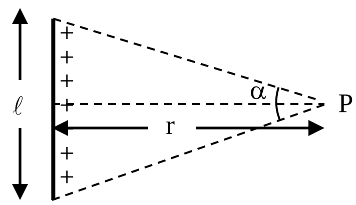
$E_{centre} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R}\sin\frac{\alpha}{2} = \frac{q}{2\pi^2\varepsilon_0 R^2}\sin\frac{\alpha}{2}$ where $\lambda = \frac{q}{R\alpha}$

For uniformly charged semi-circular ring $\alpha = 180^o$:

$E_{centre} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R} = \frac{q}{2\pi^2\varepsilon_0 R^2}$ where $\lambda = \frac{q}{\pi R}$	$V_{centre} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \frac{\lambda}{4\varepsilon_o}$



☞ **Straight uniformly charged wire or charged rod:**
λ is the linear charge density and r is the perpendicular distance of the point from the conductor.

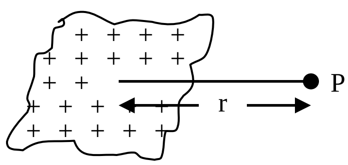


$$E_x = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda}{r}\sin\frac{\alpha}{2}$$

For infinitely long wire $\ell \to \infty$, $\alpha = \pi$

$$E_x = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda}{r} = \frac{\lambda}{2\pi\varepsilon_o r}$$

☞ **Infinite sheet of charge:**

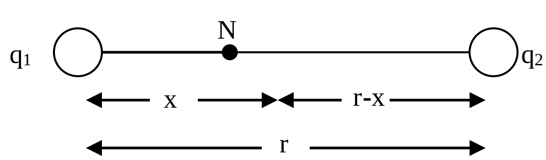
	$E = \frac{\sigma}{2\varepsilon_0}$ $(E \propto r^o)$

☞ **Electrical neutral point or null point** in an electric field is the point where resultant electric field is zero.

Charges	**Magnitude**	**Position of null point**
Like	Equal	At the midpoint of line joining the two charges.
Like	Unequal	Between the two charges and closer to the weaker charge.
Unlike	Unequal	Outside the two charges and closer to the weaker charge.
Unlike	Equal	No null point.

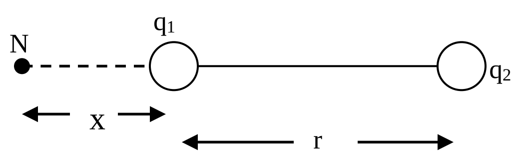
☞ Distance x of neutral point from q_1 when two charges are separated by a distance r. $(q_2 > q_1)$

$$x = \frac{r}{\sqrt{\frac{q_2}{q_1}}+1}$$ when the charges are like



when the charges are unlike

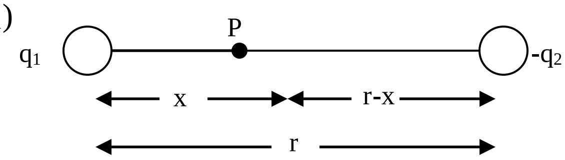
$$x = \frac{r}{\sqrt{\frac{q_2}{q_1}}-1}$$



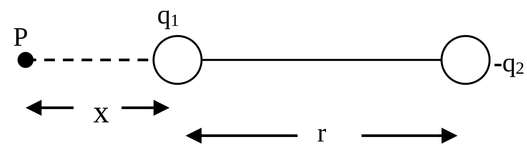
☞ **Point of zero potential between two point charges**

Nature of charges	**Magnitude**	**Position of zero potential**
Unlike	Unequal	Between the two charges and Outside the two charges closer to the weaker charge.
Unlike	Equal	At the midpoint of line joining the two charges.

For a point between the charges, $x = \dfrac{r}{\sqrt{\dfrac{q_2}{q_1}}+1}$ $(q_2 > q_1)$



For a point outside the charges, $x = \dfrac{r}{\sqrt{\dfrac{q_2}{q_1}}-1}$



Electric potential energy

☞ Potential energy stored in a charge of q_o at a point is $U = q_oV$ where V is the electric potential at that point.

☞ When a charged particle of mass m and charge q_o moves through a potential difference then decrease in electric potential energy is equal to gain in its kinetic energy

i.e. $q_o(V_A - V_B) = \frac{1}{2} m(v_B^2 - v_A^2)$

☞ **Potential energy of system of two charges in the absence of external field:**

$U = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q_1q_2}{r}$ r – distance between the two charges.

(i) Between two like charges if r increases then U decreases

(ii) Between two unlike charges if r increases then U also increases

☞ Force between two charges $F = -\dfrac{dU}{dr}$

If U is –ve then F is -ve, hence force is attractive

If U is +ve then F is +ve, hence force is repulsive

☞ P.E of system of three charges q_1, q_2 and q_3 is $U = \dfrac{1}{4\pi\varepsilon_o}\left[\dfrac{q_1q_2}{r_{12}} + \dfrac{q_2q_3}{r_{23}} + \dfrac{q_3q_1}{r_{31}}\right]$

r_{12} – distance between q_1 and q_2, r_{23} – distance between q_2 and q_3

r_{31} – distance between q_1 and q_3

☞ For system to be in equilibrium the net potential energy must be zero.

Capacitance of a conductor

☞ Capacitance $C = \dfrac{Q}{V}$ where V is the potential of the conductor and Q is the charge on it.

Capacitance of a spherical conductor

☞ Capacity of a spherical conductor of radius R is

- $C = 4\pi\varepsilon_oR$ (in free space)
- $C = 4\pi\varepsilon_o\varepsilon_rR$ (In a dielectric medium)

☞ When 'n' identical charged drops each of radius 'r', capacitance C, potential V, surface density of charge σ and energy U are combined to form a large drop then

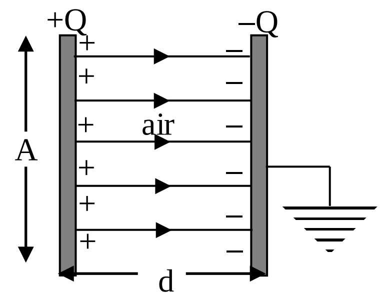
- Total charge on larger drop $Q = nq$
- Radius of larger drop $R = r\, n^{1/3}$
- Capacitance of larger drop $C' = C\, n^{1/3}$
- Electric potential of the larger drop $V' = V\, n^{2/3}$
- Surface density of larger drop $\sigma' = \sigma\, n^{1/3}$
- Energy stored in the larger drop $U' = U\, n^{5/3}$

Capacitors

☞ Dielectric constant $\varepsilon_r = \frac{C_m}{C_a}$

C_m - Capacity of a capacitor with medium as dielectric.
C_a - Capacity of a capacitor with air as dielectric.

Parallel plate capacitor



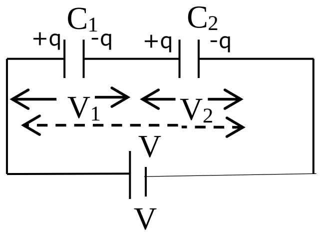
☞ A = effective overlapping area of each plate
d = separation between the plates or thickness of dielectric medium completely filled between the plates
ε_r = dielectric constant of the medium between the plates

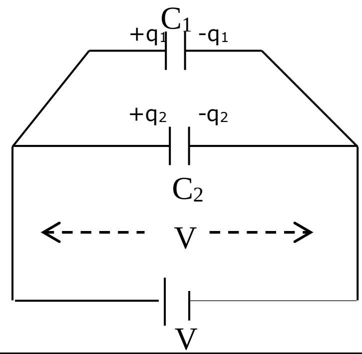
- Capacitance $C = \frac{A\varepsilon_o\varepsilon_r}{d}$ (with dielectric)
- Capacitance $C = \frac{A\varepsilon_o}{d}$ (without dielectric)
- Surface density of charge of each plate $\sigma = \frac{Q}{A}$
- Electric field due to each plates $E = \frac{\sigma}{2\varepsilon_o} = \frac{Q}{2A\varepsilon_o}$
- Net electric field between the plates $E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\varepsilon_o}$
- P.d between the plates $V = Ed = \frac{\sigma d}{\varepsilon_o} = \frac{Qd}{A\varepsilon_o}$
- Energy stored in a capacitor is $U = ½\, CV^2 = ½\, QV = ½\frac{Q^2}{C}$
- Energy density between the plates of the capacitor is $u = \frac{1}{2}\varepsilon_o E^2 = \frac{\sigma^2}{2\varepsilon_o}$ (Jm^{-3})
- Electric force of attraction between the plates $F = ½qE = ½\,\varepsilon_o E^2 A = \frac{q^2}{2\varepsilon_o A} = \frac{\sigma^2 A}{2\varepsilon_o} = \frac{CV^2}{2d}$
- Pressure on the plates = F/A = $\frac{1}{2}\varepsilon_o E^2 = \frac{\sigma^2}{2\varepsilon_o}$ = energy density

☞ Graph of Q Vs V
- straight line (Q = CV is of the form y = mx)
- slope = capacity
- area = energy stored in the capacitor.

☞ Graph of energy U Vs p.d V is a parabola ($U = ½\, CV^2$ is of the form $y = ax^2$)
Graph of energy U Vs charge q is a parabola ($U = q^2/2C$ is of the form $y = ax^2$)

Capacitors in series or parallel

☞ 



Series	Parallel
Charge on each capacitor is same	P.d across each capacitor is same
P.d across combination $V = V_1 + V_2$ $V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V$; $V_2 = \left(\frac{C_1}{C_1 + C_2}\right)V$	Total charge stored $q = q_1 + q_2$ $q_1 = \left(\frac{C_1}{C_1 + C_2}\right)q$; $q_2 = \left(\frac{C_2}{C_1 + C_2}\right)q$
Effective capacity of n capacitors $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$	Effective capacity of n capacitors $C_P = C_1 + C_2 + C_3 + \ldots + C_n$
Effective capacity is always less than least capacity of the capacitor.	Effective capacity is always greater than the highest capacity of the capacitor.
For two capacitors $C_S = \frac{C_1 C_2}{C_1 + C_2}$	For two capacitors $C_P = C_1 + C_2$
For n identical capacitors $C_S = C/n$ P.d across each $V' = \frac{V}{n}$	For n identical capacitors $C_P = nC$ Charge on each $q' = \frac{q}{n}$
P.d across each capacitor $V \propto \frac{1}{C}$ i.e. $V_1:V_2:V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$	Charge on each capacitor $q \propto C$ $q_1:q_2:q_3 = C_1 : C_2 : C_3$
Energy stored in each capacitor $U \propto \frac{1}{C}$	Energy stored in each capacitor $U \propto C$
Energy stored in the system $U_s = ½ C_S V^2$	Energy stored in the system $U_P = ½ C_P V^2$

☞ Ratio of the effective capacity of n identical capacitors first connected in parallel and then series $\frac{C_P}{C_S} = \frac{n^2}{1}$

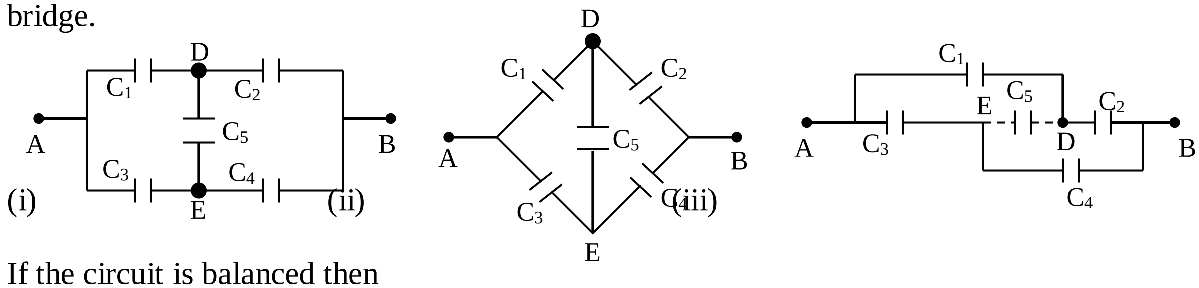
☞ **Two capacitors of capacity C_1 and C_2 are charged to a potential of V_1 and V_2 and then connected in parallel.**

	Before the combination	**After the combination**
Capacity	C_1 & C_2	C_1 & C_2
Potential difference	V_1 & V_2	Common Potential $V = \frac{C_1V_1 \pm C_2V_2}{C_1 + C_2} = \frac{Q_1 \pm Q_2}{C_1 + C_2}$
Charge	$q_1 = C_1V_1$ & $q_2 = C_2V_2$	$Q_1 = C_1V$ & $Q_2 = C_2V$
Energy	$U_1 = ½ C_1V_1^2$ & $U_2 = ½ C_2V_2^2$	$U_1' = ½ C_1V^2$ & $U_2' = ½ C_2V^2$
Loss of energy	$\Delta U = (U_1+U_2) - (U_1' + U_2') = ½ C_s(V_1 \sim V_2)^2$	

where C_s is the effective capacity of two capacitors in series

- In common potential equation, -ve sign is used when capacitors are connected together with reverse polarity.

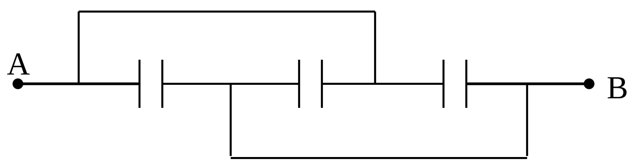
☞ **Wheatstone bridge:**
If in a network five capacitors are arranged as shown in following figure, the network is called Wheatstone bridge.



If the circuit is balanced then

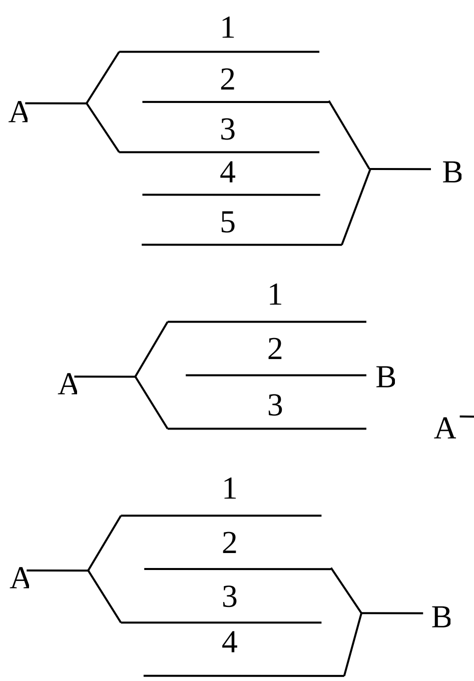
- P.d between D and E is $V_{DE} = 0$
- Charge on C_5 is $q_5 = 0$
- $\frac{C_1}{C_2} = \frac{C_3}{C_4}$
- Effective capacity between A and B is $C_{AB} = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$
- If all the capacitors are identical, each of capacity C then $C_{AB} = C$

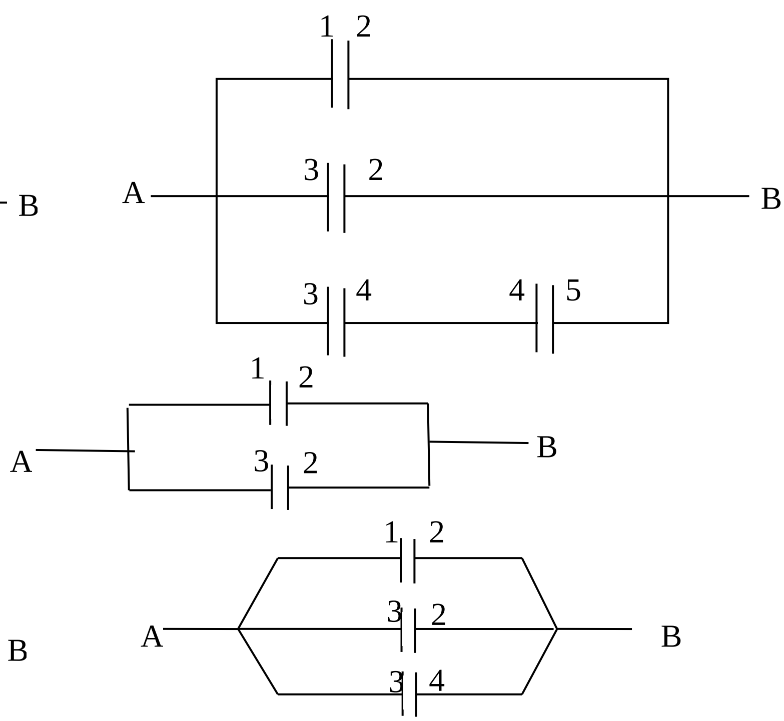
☞ If three identical capacitors each of capacity C are connected as shown in fig then $C_{eff} = 3C$, because all the capacitors are in parallel combination.



Variable capacitor:

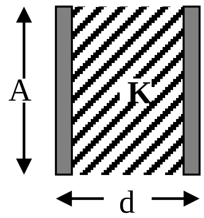
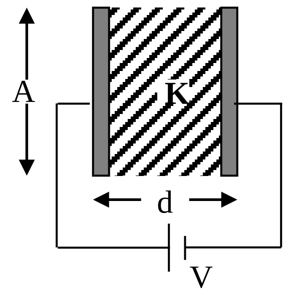
☞ **Gang capacitor:** In gang capacitor is made by stacking n equally spaced plates connected alternatively.
e.g:



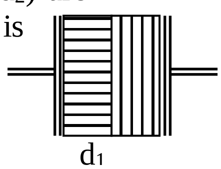


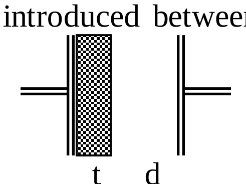
Effect of introducing a dielectric between the plates of a charged capacitor:

☞ C = capacity of charged capacitor with air as dielectric; K = dielectric constant
Q = charge; V = p.d; E = electric field; U = energy stored

Quantity	Battery is removed when dielectric is introduced	Battery remains connected when dielectric is introduced
		
Capacity	Increases; C' = KC	Increases; C' = KC
Charge	Same; Q' = Q	Increases; Q' = KQ
P.d	Decreases; V' = V/K	Same; V' = V
Electric field	Decreases E' = E/K	Same; E' = E
Energy	Decreases U' = U/K	Increases U' = UK

☞ **Combination of dielectrics**

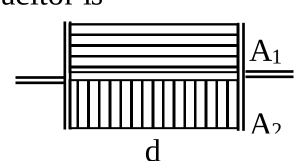
- When two dielectric slabs of same area (A) but different thickness (d_1 & d_2) are inserted between the plates of a capacitor then capacitance of such capacitor is
  $$C = \frac{\varepsilon_o A}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$
  

  where K_1 and K_2 are the dielectric constant of the medium.
- When a dielectric slab of thickness t and dielectric constant K is introduced between the plates of a capacitor then capacitance is $C' = \frac{A\varepsilon_o}{d - t + \frac{t}{K}}$
  

- When a conducting slab of thickness t is introduced between the plates of a capacitor then capacitance is
  $$C' = \frac{C}{1 - \frac{t}{d}} = \frac{A\varepsilon_o}{d - t}$$
  C = Capacitance of the capacitor before introducing the metal.

☞ When two dielectrics slabs of same thickness (d) but different areas (A_1 & A_2) are introduced between the plates of a capacitor then capacity of such a capacitor is

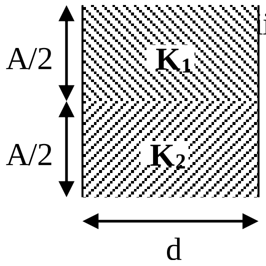
$$C = \frac{\varepsilon_o}{d}(A_1K_1 + A_2K_2)$$



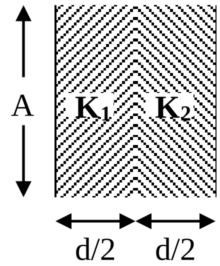
☞ **Compound dielectrics**

If several dielectric medium filled between the plates of a parallel plate capacitor in different ways as shown.

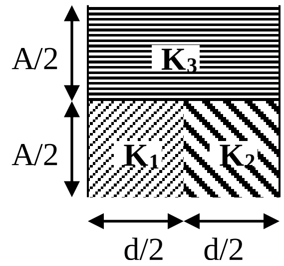
(i)



(ii)



(iii)



$$C_{eq} = \left(\frac{K_1 + K_2}{2}\right).\frac{\varepsilon_0 A}{d}$$

$$K_{eq} = \frac{K_1 + K_2}{2}$$

$$C_{eq} = \left(\frac{2K_1K_2}{K_1 + K_2}\right).\frac{\varepsilon_0 A}{d}$$

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

$$C_{eq} = \left(\frac{K_1K_2}{K_1 + K_2} + \frac{K_3}{2}\right).\frac{\varepsilon_0 A}{d}$$

$$K_{eq} = \left(\frac{K_3}{2} + \frac{K_1K_2}{K_1 + K_2}\right)$$

For n dielectrics

$$K_{eq} = \frac{K_1 + K_2 + \ldots. + K_n}{n}$$

$$\frac{n}{K_{eq}} = \left(\frac{1}{K_1} + \frac{1}{K_2} + \ldots.. + \frac{1}{K_n}\right)$$