II PUC – CHAPTER 01
ELECTRIC CHARGES AND FIELDS

Electric charges

- Total charge on a charged body is $q = \pm ne$
  where n is the number electrons lost or gained by the body.
- 6.25×10^{18} electrons make a charge of magnitude 1 C.
- Specific charge of a particle of mass m is $s = \frac{q}{m}$

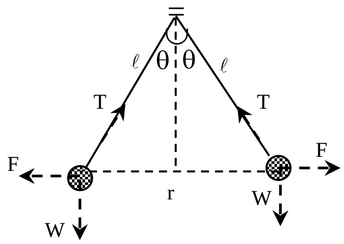
Charge density

- If a charge dq is uniformly distributed over surface area ds then surface density of charge at a point on the surface is $\sigma = \frac{dq}{ds}$ (Cm^{-2})
- Surface density of charge is inversely proportional to the radius of curvature of the surface. I.e. $\sigma \propto \frac{1}{r}$
  or $\sigma \propto$ curvature of the surface
- Surface density of a charged spherical conductor is $\sigma = \frac{q}{4\pi r^2}$
- When n identical charged drops each of radius r are combined to form a bigger drop of radius R, then surface density of the bigger drop is $\sigma' = n^{1/3}\sigma$
  where σ is the surface density of charge of each small drop $\sigma = \frac{q}{4\pi r^2}$
  Radius of the larger drop is related to the radius of the small drop by $R = n^{1/3} r$
- For two charged spherical conductors of radii R_1 and R_2 connected by a wire $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$
- Linear charge density: $\lambda = \frac{dq}{d\ell}$ (Cm^{-1}) where $d\ell$ is the length of the conductor.
- Volume charge density: $\rho = \frac{dq}{dV}$ (Cm^{-3}) where dV is the volume of the body.

Coulomb's inverse square law

- Electrostatic force between two point charges $F = \frac{Kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$ (in free space)
- $F = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2}$ (In a dielectric medium)
- If a dielectric medium of thickness t ($t < r$) and constant K is partially filled between the charges.
  The effective air separation between the charges becomes $[(r - t) + t\sqrt{K}]$.
  Hence force $F = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{[(r-t)+t\sqrt{K}]^2}$
- $\vec{F} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2}\hat{r}$ or $\vec{F} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^3}\vec{r}$ where $\hat{r}$ is the unit vector in the direction of $\vec{F}$
- Dielectric constant $\varepsilon_r = \frac{F_a}{F_m} = \frac{\varepsilon}{\varepsilon_o}$
  F_a = Force between two static point charges in free space.
  F_m = Force between same two static point charges in a dielectric medium.
- **Force per unit length between two parallel linear charged wires:** $F = \frac{1}{4\pi\varepsilon_o}\frac{2\lambda_1\lambda_2}{r}$ (Nm^{-1})
  r = separation between wires; λ_1, λ_2 = linear density of charges

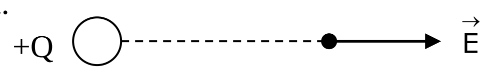
☞ **Equilibrium of suspended charged spheres:**

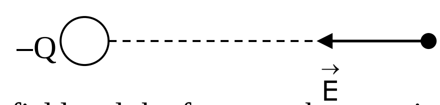


- Force between two charged spheres of mass m suspended by strings of length ℓ is
  $F = mg\tan\theta$ where θ is angle made by string with vertical
- $T\cos\theta = mg$
  $T\sin\theta = F$
  $$\tan\theta = \frac{F}{mg} = \frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2 mg}$$
- If θ is small then $\tan\theta \approx \sin\theta = \frac{r}{2\ell}$
  $$\frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2 mg} = \frac{r}{2\ell}$$
  $$r = \left[\frac{1}{4\pi\varepsilon_o}\frac{2q^2\ell}{mg}\right]^{1/3}$$

Electrostatic field

☞ **Direction of electric field:** Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge. Direction of electric field at a point is given by direction of force experienced by a unit charge placed at that point.

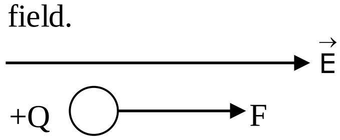


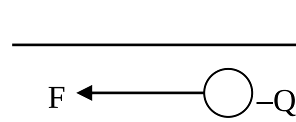


☞ The force on a positive charge is in the direction of the field and the force on the negative charge is in the opposite direction of the field.

☞ If F is the force on a charge q_o at a point then electric field is $E = \frac{F}{q_o}$ or $F = q_oE$.

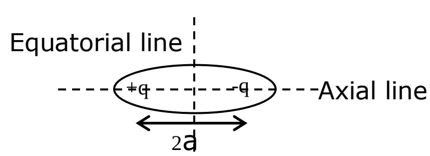
If charge is positive it experiences force in the direction of field and if it is negative direction of force is opposite to the field.





Electric dipole

☞ Dipole moment $p = q \times 2a$
q = Magnitude of both charge
'2a' = Dipole length



☞ Direction of dipole moment is from negative to positive charge.

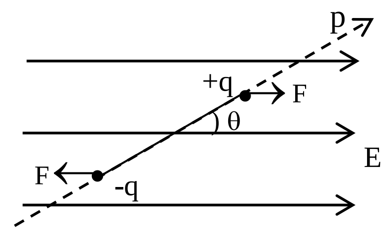
☞ **Torque on a dipole placed in a uniform electric field:**

$\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = pE\sin\theta$

$\vec{p}$ = Dipole moment;

θ = Angle between p;

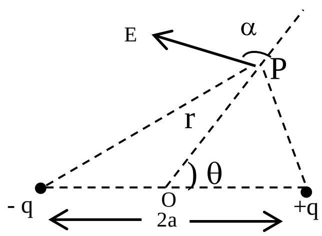
E = Uniform electric field



Special cases:

$\theta = 0^o$	$\tau_{min} = 0$	Stable equilibrium
$\theta = 90^o$	$\tau_{max} = pE\sin 90^o = pE$	-
$\theta = 180^o$	$\tau_{min} = 0$	Unstable equilibrium

☞ **Electric field and potential at any point due to a short electric dipole:**



p = Dipole moment, r = Distance between the point and the centre of the dipole.

θ - Angle made by the line joining point and the centre of the dipole with dipole axis.

where $\tan\alpha = \frac{1}{2}\tan\theta$

Electric field	Electric potential
$E = \frac{1}{4\pi\varepsilon_o}\frac{p\sqrt{3\cos^2\theta + 1}}{r^3}$	$V = \frac{1}{4\pi\varepsilon_o}\left[\frac{p\cos\theta}{r^2}\right]$
For a point on the axial line $\theta = 0^o$ $E_a = \frac{1}{4\pi\varepsilon_o}\frac{2p}{r^3}$ and $\alpha = 0$ i.e. along dipole axis in the direction of p	$V = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^2}$
For a point on the equatorial line $\theta = 90^o$ $E_e = \frac{1}{4\pi\varepsilon_o}\frac{p}{r^3}$ and $\alpha = 90^o$ i.e. parallel to dipole axis opposite to p.	V = 0

☞ Ratio of electric field at a point on the axial line and equatorial line at same distance from the centre of short dipole $E_a:E_e = 2:1$

☞ Time period of oscillation of electric dipole in uniform electric field $T = 2\pi\sqrt{\frac{I}{pE}}$

where I is the moment of inertia of the dipole about rotating axis

☞ **Work done in rotating an electric dipole in a uniform electric field:**

Total work done in rotating the dipole from equilibrium position ($\theta = 0^o$) through an angle θ will be $W = pE(1 - \cos\theta)$

Special Cases:

If $\theta = 0^o$, $W_{min} = 0$

If $\theta = 90^o$, $W = pE$

If $\theta = 180^o$, $W_{max} = 2pE$

☞ **Potential energy stored in electric dipole:**

When an electric dipole is rotated from zero energy position to the new position such that p makes an angle θ with the electric field E then potential energy stored in it is

$U = -pE\cos\theta$

Special cases:

1. When the electric dipole moment is parallel to the field, then $\theta = 0^o$

$\therefore$ U = -pE, This is the minimum potential energy.

2. When the electric dipole moment is perpendicular to the field, then $\theta = 90^o$

$\therefore$ U = 0, In this position potential energy of the dipole is zero.

3. When the electric dipole moment is antiparallel to the field, $\theta = 180^o$

$\therefore$ U = pE, This is the maximum potential energy.

Electric flux:

☞ The electric flux across any closed surface of finite area ds is $d\phi = E\cos\theta\, ds$

θ is the angle between normal to the surface and the line of force or direction of E.

☞ Electric flux is a scalar quantity i.e. $\phi = \vec{E}\cdot\vec{ds}$

☞ Its unit is Vm or Nm^2C^{-1}

Dimensional formula = $[MLT^{-2}][L^2][A^{-1}T^{-1}] = [ML^3T^{-3}A^{-1}]$

☞ Outward flux is taken as positive and inward flux is taken as negative.

☞ Electric field at a point is the electric flux per unit area around that point is $E = \frac{\phi}{A}$

Gauss theorem

☞ If ϕ is the flux across a closed surface enclosing a charge q then $\phi = \left(\frac{1}{\varepsilon_o}\right)q$

Or $\oint \vec{E}\cdot\vec{ds} = \left(\frac{1}{\varepsilon_o}\right)q$ or $\oint E ds\cos\theta = \left(\frac{1}{\varepsilon_o}\right)q$

☞ **Flux through a cube:**

Position of the charge	Centre	Face centre	Centre of edge	Corner
Flux through the cube	$\frac{q}{\varepsilon_o}$	$\frac{q}{2\varepsilon_o}$	$\frac{q}{4\varepsilon_o}$	$\frac{q}{8\varepsilon_o}$

a) Flux through each face when charge is at the centre $= \frac{q}{6\varepsilon_o}$

b) Flux through each faces (1, 2, 3) when charge is at the corner $= \frac{q}{24\varepsilon_o}$

Flux through remaining 3 faces is zero.