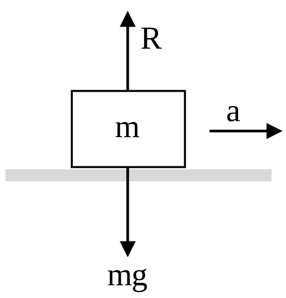
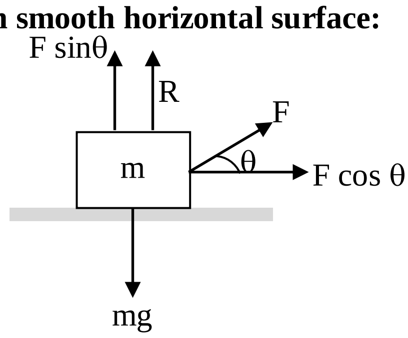
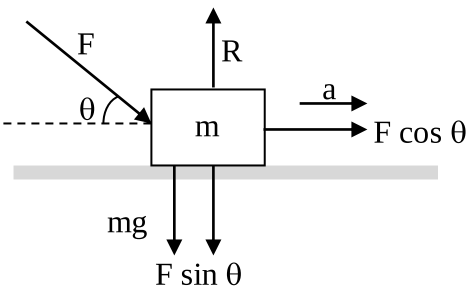
MOMENTUM AND FORCE

1) Momentum = mass × velocity i.e. p = mv
2) In a closed system consisting of a number of bodies colliding with each other, the total momentum after collision is equal to total momentum before collision.
$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$
u_1, u_2 are the velocities before collision and v_1, v_2 are the velocities after collision.
3) When $\vec{F} = 0$, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... = \text{constant}$
4) If a bullet of mass m is fired from a gun of mass M with a velocity v then recoil velocity of the gun
$V = \frac{mv}{M}$
5) Force acting on a body of mass 'm' is $F = \frac{dp}{dt} = m\frac{dv}{dt} = ma$
where a – acceleration and dp/dt = rate of change of momentum
6) Gravitational force acting on the body is F = mg, g – acceleration due to gravity
7) Gravitational force acting on the body is also called its weight W = mg
8) Impulse of a force J = F× t
where F is the average force and t is the time for which the force acts.
9) Impulse of a force = final momentum - initial momentum = mv – mu = dp
10) Area under force - time graph gives the impulse due to force.
11) Slope of momentum-time graph is equal to the force on the particle. i.e Slope $= \frac{dp}{dt} = \tan\theta = F$
12) **Acceleration of block on smooth horizontal surface:**







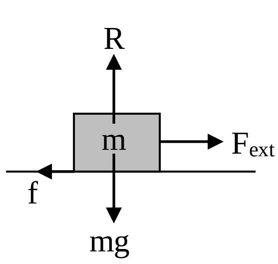
Reaction: R = mg	R = mg - Fsinθ	R = mg + Fsinθ
Acceleration: $a = \frac{F}{m}$	$a = \frac{F\cos\theta}{m}$	$a = \frac{F\cos\theta}{m}$

13) Spring force F = - k x
where k = spring constant, x = change in length
14) **Apparent weight of a man in a lift:**

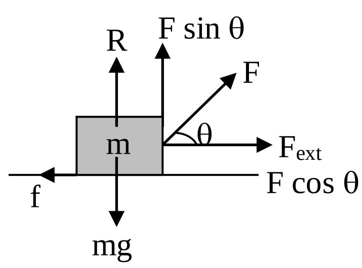
Nature of motion of lift	Apparent weight of man in lift (W_A)	Relation between apparent and real weight
At rest or moves with uniform velocity	$W_A = mg$	$W_A = W_R$
Accelerating upward	$W_A = m(g+a)$	$W_A > W_R$
Retarding upward	$W_A = m(g - a)$	$W_A < W_R$
Accelerating downward	$W_A = m(g - a)$	$W_A < W_R$
Retarding downward	$W_A = m(g + a)$	$W_A > W_R$
Falls freely	$W_A = 0$	$W_A = 0$
Accelerates downward with a > g	Man will move up and stick to the ceiling	-

FRICTION

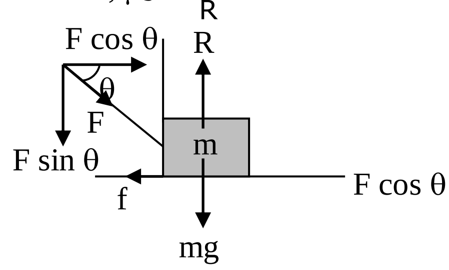
1) The force of limiting friction (F_s) is directly proportional to the normal reaction (R) between the two surfaces, i.e. $F_S \propto R$
$F_S = \mu_S R$ where μ_S is a constant called coefficient of static friction; $\mu_S = \frac{F_S}{R}$ where R is normal reaction



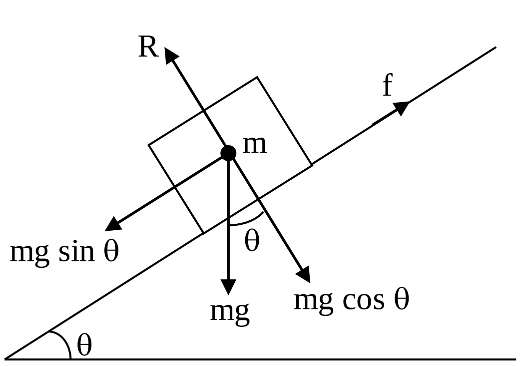
R = mg

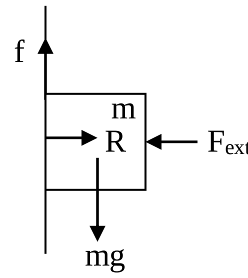


R = mg – F sin θ



R = mg + F sin θ





$R = mg\cos\theta$

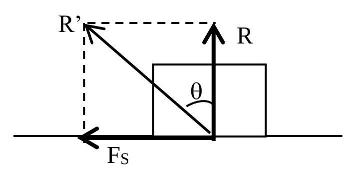
$R = F_{ext}$

2) The kinetic friction (F_K) is directly proportional to the normal reaction (R) between the two surfaces, i.e.
$F_K \propto R$
$F_K = \mu_K R$ where μ_K is a constant called coefficient of kinetic friction
$\mu_K = \frac{F_K}{R}$

3) **Motion of a block on a rough horizontal surface**
When a force F is applied along horizontal direction then net force acting on the block
$F - F_k = ma$ a – acceleration of the block of mass m
$F - \mu_K mg = ma$
For uniform velocity $F = F_K$
$F = \mu_K mg$

4) **Angle of friction:** Angle made by the resultant of normal reaction and force of limiting friction with the normal reaction is called angle of friction (θ).

$R = mg$ Normal reaction
F_S – Force of limiting friction
θ - Angle of friction
R' – Resultant of normal reaction and force of limiting friction.



From fig. $\tan\theta = \frac{F_S}{R} = \frac{\mu_S R}{R} = \mu_S$

5) The maximum angle of inclination of a surface for which a body remains at rest on that surface is called angle of repose (θ).
Or The angle of the inclined plane at which body placed on it just begins to slide is called angle of repose (θ)

6) The coefficient of static friction $\mu_S = \tan\theta$

Banking of Roads and Railway Lines

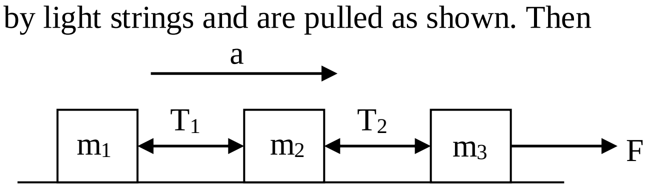
1) Maximum velocity with which a vehicle can negotiate a flat curve of radius of curvature without skidding is $v_m = \sqrt{\mu_s rg}$
a) If the vehicle moves with speed greater than v_m then it will skid and go off the road.
b) v_m is independent of mass of the vehicle.

2) Maximum speed with which a vehicle can negotiate a banked road without skidding is
$$v_m = \sqrt{\frac{(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)} rg}$$
θ = angle of banking, r = radius of curvature, μ_s = coefficient of static friction

3) For friction to be minimum $\mu_s \approx 0$, then $v_m = \sqrt{rg\tan\theta}$
or Angle of banking $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$
Driving at this speed on a banked road will cause no wear and tear of the tyres.

FREE BODY DIAGRAM (FBD)

1. Masses m_1, m_2, m_3 are connected by light strings and are pulled as shown. Then
For m_1 : $T_1 = m_1 a$
For m_2 : $T_2 - T_1 = m_2 a$
For m_3 : $F - T_2 = m_3 a$



a) Acceleration of the system $a = \frac{F}{(m_1 + m_2 + m_3)}$
b) $T_1 = m_1 a$
c) $T_2 = (m_1 + m_2)a$
(T_1 and T_2 are tensions in the strings)
In the above case if μ is coefficient of kinetic friction between any body and the horizontal surface;
for m_1 : $T_1 - \mu m_1 g = m_1 a$
for m_2 : $T_2 - T_1 - \mu m_2 g = m_2 a$

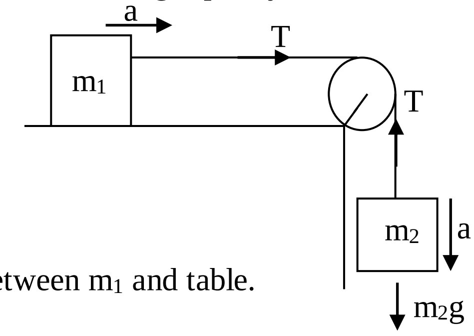
for m_3 : $F - T_2 - \mu m_3 g = m_3 a$

2. Mass m_1 is on a smooth table. A string attached to it passes over a light pulley and carries m_2.

for m_1 : $T = m_1 a$

for m_2 : $m_2 g - T = m_2 a$

Acceleration of the system $a = \frac{m_2 g}{m_1 + m_2}$



Tension in the string $T = \frac{m_1 m_2 g}{(m_1 + m_2)}$

Thrust on pulley = $\sqrt{2}\,T$

In the above case if μ is coefficient of kinetic friction between m_1 and table.

for m_1 : $T - \mu m_1 g = m_1 a$

for m_2 : $m_2 g - T = m_2 a$

$\Rightarrow a = \frac{(m_2 - \mu m_1)g}{m_1 + m_2}$

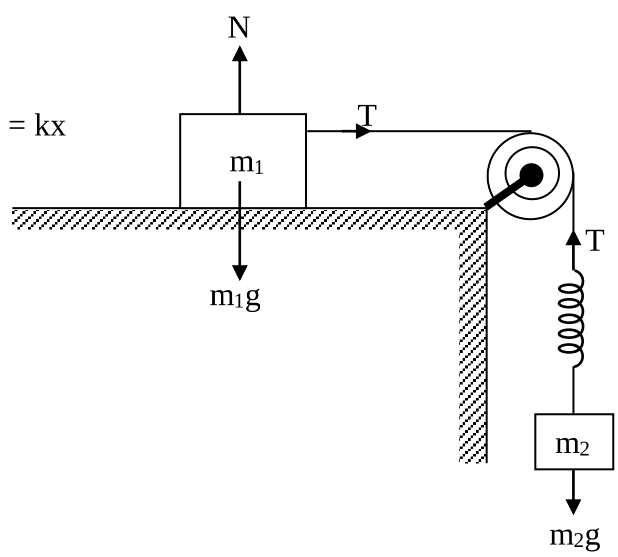
$T = m_2(g + a)$

3. **Pulley-spring system (at steady state)**

Tension $T = \frac{m_1 m_2 g}{(m_1 + m_2)}$

If x is the extension in the spring then $T = kx$

$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1 + m_2)}$



4. When two bodies of masses m_1 and m_2 are connected by a light string passing over a light pulley, ($m_2 > m_1$)

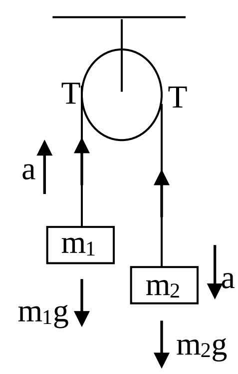
for m_1 : $T - m_1 g = m_1 a$

for m_2 : $m_2 g - T = m_2 a$

Acceleration of each body $a = \frac{(m_2 - m_1)g}{m_1 + m_2}$

$T = \frac{2 m_1 m_2 g}{(m_1 + m_2)}$

Thrust on the pulley = 2T



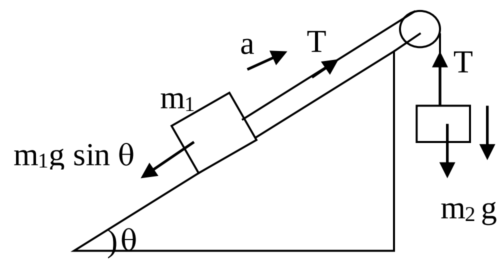
5. A body of mass m_1 is on a smooth inclined plane as shown. A string attached to it passes over a light pulley and carries mass m_2 at the other end as shown.

If m_1 moves up and m_2 moves down

for m_1 : $T - m_1 g\sin\theta = m_1 a$

for m_2 : $m_2 g - T = m_2 a$

$\Rightarrow$ Acceleration of each block is $a = \frac{(m_2 - m_1 \sin\theta)g}{(m_1 + m_2)}$



Tension in the string $T = m_2(g - a)$

Thrust on the pulley = $\sqrt{T^2 + T^2 + 2T^2\cos(90-\theta)} = \sqrt{2T^2(1+\sin\theta)}$

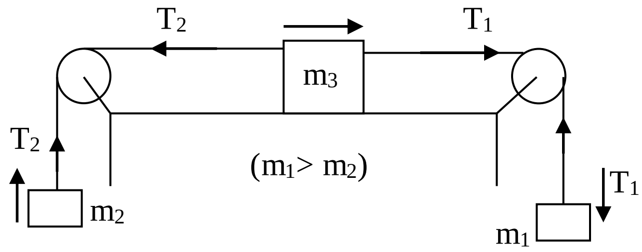
If μ_k if the coefficient of kinetic friction between m_1 and inclined surface in the above case

for m_1 : $T - (m_1 g\sin\theta + \mu_k m g\cos\theta) = m_1 a$

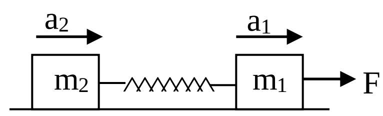
for m_2 : $m_2 g - T = m_2 a$

6. A system of three blocks is connected by two strings as shown (no friction)

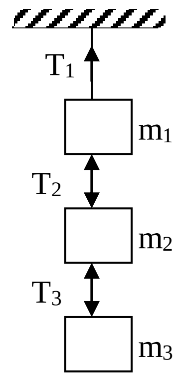
for m_1 : $m_1g - T_1 = m_1a$
for m_2 : $T_2 - m_2g = m_2a$
for m_3 : $T_1 - T_2 = = m_3a$
$\Rightarrow$ Acceleration of the system is
$a = \frac{(m_1 - m_2)g}{(m_1 + m_2 + m_3)}$



7. If two bodies are connected by a spring and the system is pulled as shown, then accelerations are different
For m_1: $F - f = m_1a_1$
For m_2: $f = m_2a_2$
(f is spring force)

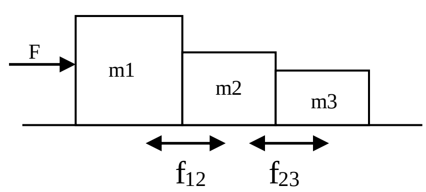


8. Consider three blocks of different masses m_1, m_2 and m_3 connected by strings as shown. The system is at rest and for different masses
for m_3 : $T_3 = m_3g$
for m_2 : $T_2 - T_3 = m_2g$
$\Rightarrow T_2 = (m_2 + m_3)g$
for m_1 : $T_1 - T_2 = m_1g$
$\Rightarrow T_1 = (m_1 + m_2 + m_3)g$



If the system moves up with uniform acceleration 'a' in the above case,
$T_1 = (m_1 + m_2 + m_3)(g + a)$
$T_2 = (m_2 + m_3)(g + a)$
$T_3 = m_3(g + a)$
If the system moves down with uniform acceleration 'a', it will be '-a' in the place of 'a'.

9. A system consists of the bodies of different masses as shown which are in contact. A force F is applied as shown. All the bodies move with the same acceleration 'a'.
for m_1 : $F - f_{12} = m_1a$
for m_2 : $f_{12} - f_{23} = m_2a$
for m_3 : $f_{23} = m_3a$
$a = \frac{F}{(m_1 + m_2 + m_3)}$



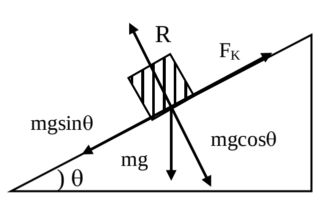
Contact force between m_1 and m_2 is $f_{12} = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$

Contact force between m_2 and m_3 is $f_{23} = \frac{m_3F}{(m_1 + m_2 + m_3)}$

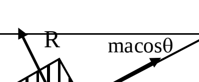
In the above case, if there is friction between the blocks and horizontal surface.
for m_1 : $F - f_{12} - \mu_1 m_1 g = m_1a$
for m_2 : $f_{12} - f_{23} - \mu_2 m_2 g = m_2a$
for m_3 : $f_{23} - \mu_3 m_3 g = m_3a$

10. Acceleration of a body on smooth inclined plane
Normal reaction $R = mg\cos\theta$
Acceleration down the inclined plane $a = g\sin\theta$
Force along inclined plane $F = mg\sin\theta$
Velocity at the bottom of the inclined plane $v = \sqrt{2gh} = \sqrt{2g\ell\sin\theta}$
h = height of the inclined plane of length ℓ
Time taken to reach the bottom $t = \sqrt{\frac{2\ell}{g\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

11. Acceleration of a body on rough inclined plane:
Normal reaction $R = mg\cos\theta$
Force due to gravity $= mg\sin\theta$
Acceleration of body $a = g(\sin\theta - \mu_k\cos\theta)$
Force on body $F = mg(\sin\theta - \mu_k\cos\theta)$



12. When the inclined plane is given a horizontal acceleration 'a' on smooth inclined plane



Normal reaction R = mgcosθ + masinθ
Force on body F = mgsinθ - macosθ
Acceleration of body a' = gsinθ - acosθ
Force on body F = mgsinθ - macosθ
(i) If mgsinθ > macosθ, then body accelerates downward
(ii) If mgsinθ = macosθ or a = g tanθ then body remain at rest relative to the inclined plane
(iii) If mgsinθ < macosθ, then body accelerates upward

13. Maximum length of a chain which can hang from the table without sliding is

$\ell = \frac{\mu L}{1 + \mu}$ L = length of chain; μ = coefficient of static friction

****