CHAPTER 08 - GRAVITATION

Newton's Law of Gravitation

1) $F = G\frac{m_1 m_2}{r^2}$

F is the force between two objects of mass m_1 and m_2.
r is the distance between their centres, G is the universal gravitational constant.

2) **Vector form:** Gravitational force exerted by m_1 on m_2 is $\vec{F} = G\frac{m_1 m_2}{r^2}\left(-\hat{r}\right)$

$\hat{r}$ is the unit vector directed from m_1 to m_2. Negative sign shows that gravitational force is attractive in nature.

3) $G = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}][L]^2}{[M^2]}$

$[G] = [M^{-1}L^3 T^{-2}]$

Acceleration due to gravity (g)

1) Acceleration due to gravity on or near the surface of earth $g = \frac{GM}{R^2} = \frac{4\pi RGd}{3}$

M = mass of the earth, R = radius of the earth, d = density of the earth

2) Above expression is used to find the acceleration due to gravity very close to the surface of any planet.

3) The value of g is independent of the mass of the body but depends on the mass and radius of the planet.

Variation of acceleration due to gravity

1) **Variation with altitude (height):**

a) Acceleration due to gravity at a height h above the surface of the earth, $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{GM}{(R+h)^2}$

R - Radius of the earth, g – acceleration due to gravity on the surface of the earth, G – Gravitational constant

b) If h << R then $g' = g\left[1 - \frac{2h}{R}\right]$

c) Decrease in the value g on going up a height 'h' above the surface of earth $\Delta g = \frac{2gh}{R}$

2) **Variation of g with depth:**

a) Acceleration due to gravity at a depth h below the surface of the earth, $g' = g\left(1 - \frac{h}{R}\right)$

b) Decrease in the value g at a depth h below the surface of the earth $\Delta g = \frac{gh}{R}$

c) Change in g at a height h above the surface of the earth is twice the change in g at a depth h below the surface of the earth.

3) **Variation of g with latitude:**

a) Acceleration due to gravity at latitude θ is $g' = g - R\omega^2 \cos^2\theta$ where ω is the angular velocity of the earth.

g is the acceleration due to gravity at latitude θ if earth were to be at rest.

At the equator $\theta = 0^o \therefore g' = g - R\omega^2$

At the poles $\theta = 90^o \therefore g' = g$

The orbital velocity of the satellite

1) $v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{gR^2}{r}}$ where r = R + h

M is the mass of the earth, R is the radius of the earth and h is the height of satellite from the earth's surface, r is the distance between the satellite and the centre of the earth.

2) When h << R then satellite is close to the planet, $v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

3) Kinetic energy of satellite of mass m is $K = \frac{mGM}{2r} = \frac{mgR^2}{2r}$

4) Kinetic energy of mass m very close to the earth is $K = \frac{mGM}{2R} = \frac{mgR}{2}$

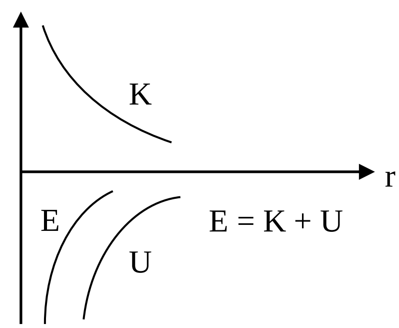
5) Potential energy $U = -\frac{mGM}{r} = -\frac{mgR^2}{r}$

6) Gravitational potential at the same point is $V = -\frac{GM}{r} = -\frac{gR^2}{r}$

7) Potential energy of mass m very close to the earth is $U = -\frac{mGM}{R} = -mgR$

8) The total energy is, $E = K + U = -\frac{mGM}{2r} = -\frac{mgR^2}{2r}$

9) Total energy of a mass m on the earth is $U = -\frac{mGM}{R} = -mgR$



Period of revolution of the satellite

1) $T = 2\pi\left(\frac{R+h}{v_o}\right) = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

2) When h << R, i.e. when the satellite is very close the planet then $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$

3) Orbital angular velocity of a satellite $\omega = \sqrt{\frac{g}{R}}$

Escape Velocity

1) Escape velocity of a body at an altitude h is $v_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2gR^2}{R+h}}$

2) Escape velocity of a body on the surface of the earth is $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

where M = mass of the earth; R = radius of the earth.

3) Escape velocity and orbital velocity are related by $v_e = \sqrt{2}v_o$

4) Kinetic energy of escape = 2 × kinetic energy of orbital motion.

Kepler's Laws of Planetary Motion

1) **I law (Law or orbits):** Every planet moves in an elliptical orbit around the sun with sun at one of the focus.

2) **II law (Law of areas):** The line joining the planet to the sun sweeps out equal areas in equal interval of time. i.e. areal velocity is constant

This law shows that planet moves faster when it is closer to the sun and slower when it is away from the sun such that **r × v = constant** (i.e. Angular momentum of a planet is conserved)
where r is the distance of the planet from the sun and v is the corresponding velocity.

Areal velocity = $\frac{dA}{dt} = \frac{r^2\omega}{2} = \frac{L}{2m}$

where L is the angular momentum of the planet of mass m in the given orbit.
ω - angular velocity.

3) **III law (Law of periods):** The square of period of revolution of any planet is directly proportional to the cube of the semimajor axis of the elliptical orbit.
According to this law $T^2 \propto r^3$, where T is the period of revolution of the planet and 'r' is average distance of the planet from the sun.

Gravitational field and Gravitational potential

1) If F is the gravitational force experienced by a mass m then gravitational intensity E = F/m
2) If W is the work done in moving a mass m from infinity to a point in a gravitational field then gravitational potential V = W/m.
3) Work done in moving a body from a point in gravitational field to infinity against gravitational field is $W = \frac{GMm}{r}$ where r = R + h
4) Work done in moving a body from surface of the earth to infinity against gravitational field is $W = \frac{GMm}{R}$ = mgR ∵ $GM = gR^2$

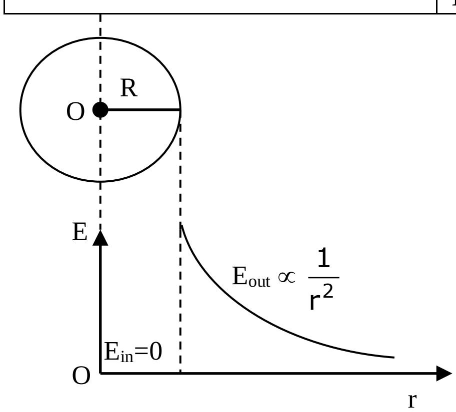
Gravitational field and Gravitational potential due to a system

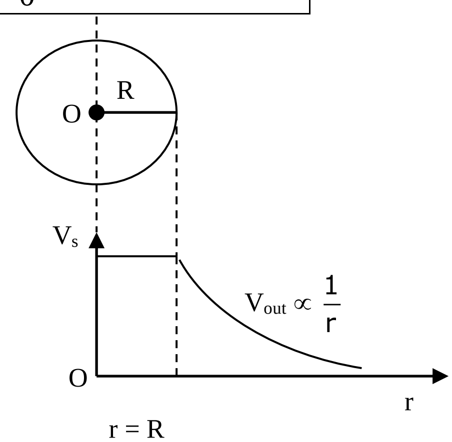
1) **Point mass:**

$E = \frac{GM}{r^2}$ or $\vec{E} = \frac{GM}{r^2}\hat{r}$	$V = -\frac{GM}{r}$

2) **Spherical shell of radius R:**
r = distance of the point from the centre of the sphere

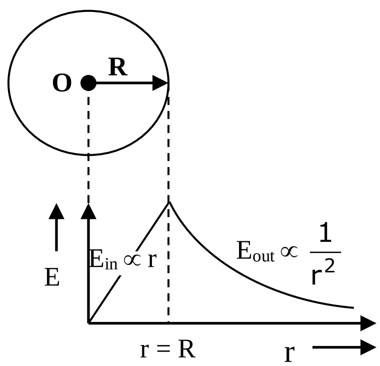
$E_{outside} = \frac{GM}{r^2}$	$V_{outside} = -\frac{GM}{r}$
$E_{surface} = \frac{GM}{R^2}$	$V_{surface} = -\frac{GM}{R}$
$E_{inside} = 0$	$V_{inside} = V_{surface}$ $P.d_{inside} = 0$

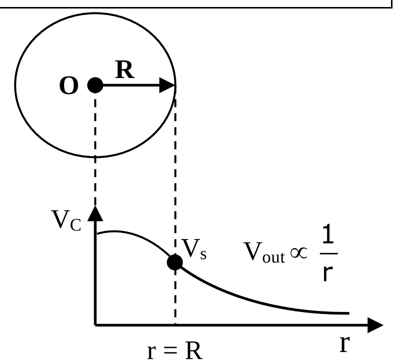




3) **Solid sphere of radius R:**

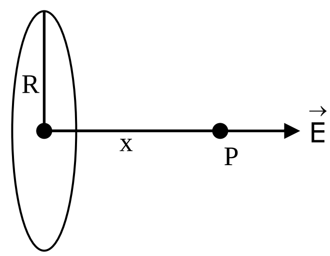
$E_{outside} = \frac{GM}{r^2}$	$V_{outside} = -\frac{GM}{r}$
$E_{surface} = \frac{GM}{R^2}$	$V_{surface} = -\frac{GM}{R}$
$E_{inside} = \frac{GMr}{R^3}$	$V_{inside} = -\frac{GM\left(3R^2 - r^2\right)}{2R^3}$
$E_{centre} = 0 \quad (\because r = 0)$	$V_{centre} = -\frac{3GM}{2R} = -\frac{3}{2}V_{surface}$





4) **Ring:**

x is the distance of the point from the centre of the ring. 'R' is the radius of the ring.



$E_{axis} = \frac{GMx}{(R^2 + x^2)^{3/2}}$	$V_{axis} = -\frac{GM}{\sqrt{x^2 + R^2}}$
At $x >> R$, $E = \frac{GM}{x^2}$	At $x >> R$, $V = \frac{GM}{x}$
At $x = 0$, $E_{centre} = 0$	$V_{centre} = -\frac{GM}{R}$
If $x = \frac{R}{\sqrt{2}}$, The maximum value is $E_{max} = \frac{2GM}{3\sqrt{3}R^2}$	$V_{max} = -\frac{2GM}{\sqrt{6}R}$

5) **Gravitational Potential Energy of a two particle system:**
The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by, $U = -\frac{Gm_1m_2}{r}$

6) **Gravitational potential energy for a system of particles:**
The gravitational potential energy for a system of particles (say m_1, m_2, m_3)

$$U = -G\left[\frac{m_1m_2}{r_{12}} + \frac{m_2m_3}{r_{23}} + \frac{m_3m_1}{r_{31}}\right]$$