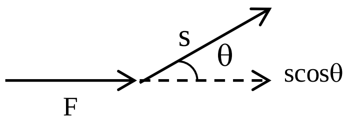
CHAPTER 06 – WORK, ENERGY AND POWER

Work

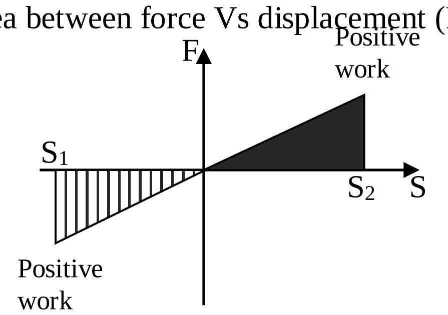
1) **Work done by a constant force:**
Work is said to be done when the point of application of force is displaced.

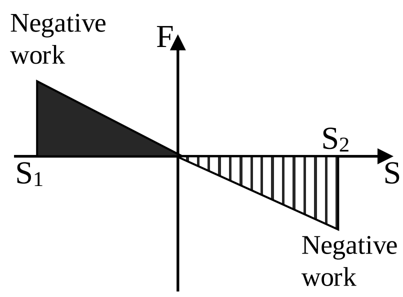


If the displacement s takes place at an angle θ with the direction of the force, then work done on the body is $W = Fs\cos\theta$

2) Work is the dot product of force and displacement i.e. $W = \vec{F} \cdot \vec{s} = Fs\cos\theta$

3) Area between force Vs displacement (F-s) graph gives the work done.





4) **Work done by a variable force:**

Work done when a body is moved from positions s_1 to s_2 is $W = \int_{s_1}^{s_2} \vec{F} \cdot \vec{ds} = \int_{s_1}^{s_2} F ds\cos\theta$

5) **Cartesian Form:** $\vec{r}_1$ and $\vec{r}_2$ be the position vectors then total work done $W = \int_{r_1}^{r_2} \vec{F} \cdot \vec{dr}$

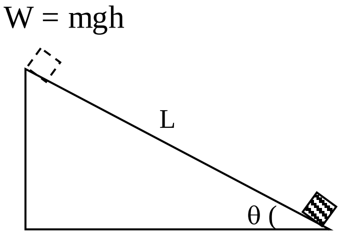
$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{r} = \vec{r}_2 - \vec{r}_1 = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$W = \int_{r_1}^{r_2} \vec{F} \cdot \vec{dr} = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

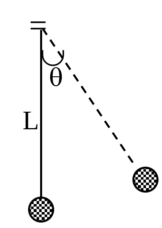
Work done interms of position vectors $W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$

6) Work done in lifting a body of mass m through a height h above the is W = mgh

7) Work done in moving a body of mass m through a distance L along an frictionless inclined plane
$W = mgL\sin\theta$, where θ is the angle made by the plane with horizontal. (L = length of plane)



8) Work done in pulling the bob of a simple pendulum of length L so that it makes an angle θ with the vertical,
$W = mgL(1 - \cos\theta)$, m – mass of the bob.



9) Work done by external force in stretching a spring by length x is $W = \frac{1}{2} kx^2$
where k is spring constant or force constant
Work done by spring force is $W = -\frac{1}{2} kx^2$

10) Work done in stretching a spring from a length x_1 to x_2 is $W = \frac{1}{2} k(x_1^2 - x_2^2)$

11) **Gravitational potential energy** of a body of mass 'm' at a height 'h' from the surface of the earth is $U_g = mgh$, where 'g' is the acceleration due to gravity at the place.

12) **Elastic potential energy of a spring:** $U = \frac{1}{2} kx^2 = \frac{1}{2} Fx = \frac{F^2}{2k}$
where k = spring constant x = change in length, F = force applied on the spring.

13) **Relation between conservative force and potential energy:**
$$\vec{F} = -\nabla U = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

14) P.E is positive, if force is repulsive and P.E is negative, if force is attractive in nature

15) For regularly shaped uniform bodies, the potential energy change can be calculated by considering their mass to be centred at the geometrical centre point.
e.g. For a uniform vertical rod of length L, the P.E = mgL/2

16) Kinetic energy of a body of mass m having velocity 'v' is $K = \frac{1}{2} mv^2$

17) Kinetic energy of a body $K = \frac{1}{2} mv^2 = \frac{1}{2} pv = \frac{p^2}{2m}$ p = mv
Linear Momentum $p = \sqrt{2mK} = \frac{2K}{v}$

18) i.e. $W = \Delta K = \frac{1}{2} [m(v^2 - u^2)]$ where m = mass, v = final velocity, u – initial velocity,

19) If the extreme position of the bob of a simple pendulum is at a height h above the mean position, then the bob will cross the mean position with velocity $v = \sqrt{2gh}$

20) $K = \frac{p^2}{2m}$. This equation is of the form $y = ax^2$ which represent a parabola.

$\therefore$ Variation of kinetic energy (K) with momentum (p) of a body is a parabola. (fig 1)

$\sqrt{K} = \frac{p}{\sqrt{2m}}$ This equation is of the form y = mx which represent a straight line.

$\therefore$ Variation of $\sqrt{K}$ with p is a straight line. (fig 2)

Variation of $\sqrt{K}$ with $\frac{1}{p}$ is a hyperbola. (fig 3)

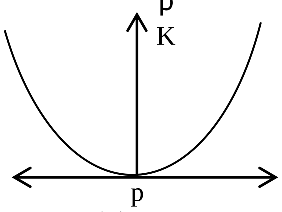


Fig (1)

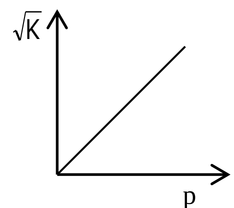


Fig (2)

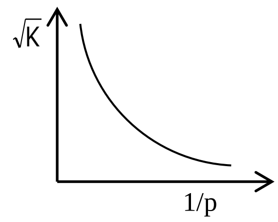


Fig (3)

21) **Translatory equilibrium:**

At points where $\frac{dU}{dx} = 0$, F = 0 then body will be in translatory equilibrium.

(i) If a particle slightly displaced from its equilibrium position, tends to come back to the same position then it is said to be in stable equilibrium.
P.E of the body is minimum in stable equilibrium
(ii) If a particle slightly displaced from its equilibrium position, tends to go away from that position then it is said to be in unstable equilibrium.
P.E of the body is maximum in unstable equilibrium
(iii) A body is said to be in neutral equilibrium if on being slightly displaced, it remains in the new position.
P.E of the body is constant in neutral equilibrium

22) Power = $\frac{\text{work}}{\text{time}}$ i.e. $P = \frac{W}{t}$

23) Average power $P_{av} = \frac{\Delta W}{\Delta t}$

24) Instantaneous power $P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$

where dW is the small amount of work done in small interval of time dt.

25) Power of an agent at any instant is equal to the dot product of the applied force and the velocity at that instant. I.e. $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$ θ - angle between F and v

26) Area under force- velocity (F-v) graph is equal to power dissipated.

27) Area under power- time (P-t) graph is equal to work done.

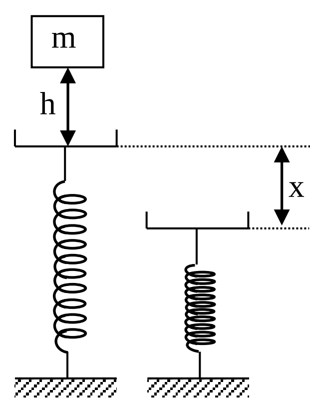
28) 1 horsepower (HP) = 746 watt

29) Efficiency of a device = $\frac{\text{Outputpower}}{\text{Inputpower}} = \frac{\text{Powerused}}{\text{RealPower}}$

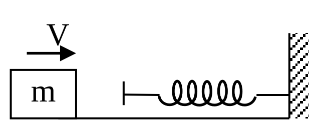
Law of conservation of energy

1) Total energy of a freely falling body of mass m dropped from a height h is = mgh

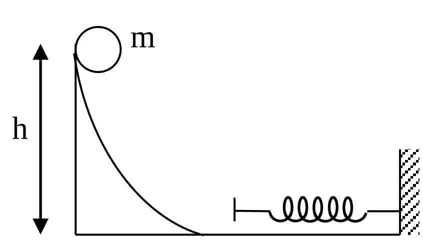
2) A body of mass m is freely released from a height h above a vertical spring. If the spring gets compressed by x then $mg(h + x) = \frac{1}{2} kx^2$



3) A body of mass m moving with a speed v collides with a horizontal spring. If x_{max} is the compression of the spring then $\frac{1}{2} mv^2 = \frac{1}{2} kx^2_{max}$



4) A body of mass m released from a height h collides with a spring. If x_{max} is the compression of the spring then $mgh = \frac{1}{2} kx^2_{max}$



Collisions

1) For perfectly elastic collision, relative velocity before collision is equal to relative velocity after the collision
i.e. $u_1 - u_2 = v_2 - v_1$

2) If two bodies stick together after collision or move together with the common velocity then collision is called **perfectly inelastic collision.** i.e. $v_1 = v_2$

3) **Equations for elastic collision and inelastic collision:**
m_1 and m_2 masses of two bodies. u_1 and u_2 velocities before collision and v_1 and v_2 velocities after collision
(a) For elastic collision total momentum is conserved.
i.e. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Total kinetic energy is also conserved. $\therefore \frac{1}{2} m_1u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
(b) For inelastic collision total momentum is conserved.
i.e. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
But kinetic energy is not conserved. $\therefore \frac{1}{2} m_1u_1^2 + \frac{1}{2}m_2 u_2^2 > \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
Loss of kinetic energy = $\left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \left[\frac{1}{2} m_1v_1^2 + \frac{1}{2}m_2 v_2^2\right]$

4) Velocities of colliding particles after the collision in one dimensional (head–on) elastic collision:
Velocity of 1st body after collision $v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2u_2}{m_1 + m_2}$
Velocity of 2nd body after collision $v_2 = \frac{m_2 - m_1}{m_2 + m_1}u_2 + \frac{2m_1u_1}{m_2 + m_1}$
Special cases:
Case (i): If the two masses are equal i.e. $m_1 = m_2 = m$
Then $v_1 = u_2$ and $v_2 = u_1$
That is, initial velocity of one object becomes the final velocity of the other. The objects interchange their velocities.
Case (ii): If a light object m_1 collides with a big object m_2 initially at rest, ($m_1 << m_2$, $u_2 = 0$), then $m_1 - m_2 \approx -m_2$ and $m_1 + m_2 \approx m_2$.
Then $v_1 = -u_1$ and $v_2 \approx 0$ by assuming that $2m_1u_1 << m_2 + m_1$
i.e. The lighter object rebounds with almost same speed. The heavier object gets negligible velocity.
Case (iii): If a heavier object in motion collides with a lighter object at rest, ($m_1 >> m_2$, $u_2 = 0$), then $m_1 - m_2 \approx m_1$ and $m_1 + m_2 \approx m_1$.
Then $v_1 = u_1$ and $v_2 = 2u_1$
That is, the colliding heavier object moves without change in its velocity, whereas the lighter object, which is initially at rest starts moving with double the initial velocity of the bigger object.

5) Loss of kinetic energy in a perfectly inelastic collision in one dimension. ($v_1 = v_2$)
$\Delta E = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2$

6) If u_1, u_2 are the velocities of two bodies before collision and v_1, v_2 are the velocities after collision then coefficient of restitution
$e = \left[\frac{v_2 - v_1}{u_1 - u_2}\right] = \frac{\text{relative velocity of seperation after collision}}{\text{relativeve velocity of approach before collision}}$
Value of e lies between 0 to 1.
e = 1 for perfectly elastic collision
e =0 for perfectly inelastic collision
0 < e < 1 for inelastic collision.

7) When a rubber ball is dropped $e = \sqrt{\frac{h_2}{h_1}} = \frac{v_2}{v_1}$
where h_1 and h_2 are initial and final height of the ball before and after the rebounce from the ground.
v_1 is the velocity with which rubber ball hits the floor and v_2 velocity with which ball rebounces.