CHAPTER 03 - MOTION ALONG A STRAIGHT LINE

1) A particle moves along the circumference of a circle of radius 'r'. If the angle made by the radius vector at the centre is θ

Displacement = $2r\sin\left(\frac{\theta}{2}\right)$

Distance traveled = $r\theta$

2) If $\vec{s}_1, \vec{s}_2, \vec{s}_3....$ are the displacement of a body then net displacement $\vec{s} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3....$

3) Average speed v = $\frac{\text{Total distance}}{\text{Total time}} = \frac{\Delta s}{\Delta t}$

4) For uniform velocity v = $\frac{s}{t}$ where s is the distance travelled in time t second.

5) Instantaneous velocity v = $\frac{ds}{dt}$ where ds is the displacement in a time dt

6) Average velocity v_{av} = $\frac{\text{Total displacement}}{\text{Total time}}$

Also $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$

where $\vec{r}_1$ is the position of the body at time t_1 and $\vec{r}_2$ its position at time t_2

Displacement = $\vec{r}_2 - \vec{r}_1$

7) If a body moves with speeds v_1, v_2, v_3……v_n in different time intervals t_1, t_2, t_3….. then average speed of the body $v_{av} = \frac{s_1 + s_2 + s_3 + ... + s_n}{t_1 + t_2 + t_3 + ... + t_n} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + ...}{t_1 + t_2 + t_3 + ...}$

If time intervals are same, then, $v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_n}{n}$ (arithmetic mean)

For two equal intervals of time (n = 2), $v_{av} = \frac{v_1 + v_2}{2}$

For three equal intervals of time (n = 3), $v_{av} = \frac{v_1 + v_2 + v_3}{3}$

8) If a body travels distances s_1, s_2, s_3….with speeds v_1, v_2, v_3… then

average speed of the body $v_{av} = \frac{s_1 + s_2 + s_3 + ... + s_n}{t_1 + t_2 + t_3 + ... + t_n} = \frac{s_1 + s_2 + s_3 + ... + s_n}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} ... + \frac{s_n}{v_n}}$

If distances are equal, $v_{av} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} ... + \frac{1}{v_n}}$ (harmonic mean)

For two equal distances (n =2), $v_{av} = \frac{2v_1v_2}{v_1 + v_2}$

For three equal distances (n = 3), $v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

9) If velocity varies with time then average velocity $\langle v \rangle = \frac{\int v\,dt}{\int dt}$

10) Acceleration a = $\frac{v-u}{t} = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

u – initial velocity, v - final velocity, t is the time in which velocity changes.

11) Instantaneous acceleration a = $\frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$

where dv is the change in velocity in a time dt

12) Average acceleration a_{av} = $\frac{\text{change in velocity}}{\text{Total time}}$

Also $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

where $\vec{v}_1$ is the velocity of the body at time t_1 and $\vec{v}_2$ its velocity at time t_2

13) If a body travels with uniform acceleration a_1 in time t_1 and with uniform acceleration a_2 for time t_2 then average acceleration of the body is $a_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$

14) u – initial velocity and v – final velocity, s – displacement
a – uniform acceleration, t – time
(i) Velocity of a body in a time t is v = u + at **or** $\vec{v} = \vec{u} + \vec{a}t$
or $v = \int a\,dt$
(ii) Distance travelled by the body in time t is $s = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2 = \left(\frac{u+v}{2}\right)t$
or $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2 = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$ **or** $s = \int v\,dt$
(iii) Velocity at the end of distance s is $v^2 = u^2 + 2as$ **or** $\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{s}$
(iv) Also average velocity $v_{av} = \frac{u+v}{2}$

15) Distance travelled during n^{th} second of motion $s_n = u + a(n - \frac{1}{2}) = u + \frac{a}{2}(2n-1)$

16) Uniform acceleration $a = \frac{s_2 - s_1}{t^2}$ where s_1 and s_2 are distances travelled in consecutive intervals of time t each.

17) A body moving with a velocity u is stopped by a force after covering a distance s.
In $v^2 = u^2 + 2as$, v = 0 and a is negative and constant
$s = -\frac{u^2}{2a}$
$s \propto u^2$ or stopping distance $\propto$ (initial velocity)^2

MOTION OF A BODY MOVING UNDER GRAVITY

1) **Equations of motion**
$v = u \pm gt$
$s = ut \pm \frac{1}{2}gt^2$
$v^2 = u^2 \pm 2gs$
$s_n = u \pm g(n - \frac{1}{2}) = u \pm \frac{g}{2}(2n-1)$
$\pm g = \frac{s_2 - s_1}{t^2}$

2) **Body thrown vertically upward**
a) velocity at the highest point is zero (v = 0)
b) acceleration a = - g
c) velocity and acceleration are in opposite directions.
d) Maximum height reached $h_{max} = \frac{u^2}{2g}$ or $h \propto u^2$
Time of ascent $t_a = u/g = \sqrt{\frac{2h_{max}}{g}}$
Time of flight $t_f = 2u/g$
Time of ascent = time of decent
e) Under air resistance time of ascent is lesser than time of decent ($t_a < t_d$)
f) When the particle reaches the point of projection it has the same magnitude of velocity as the velocity of projection, but in the opposite direction.
g) In vertical motion under gravity the velocity at any point of its path is same in magnitude, when it is going up and coming down.

3) **Body dropped from a certain height**
a) Initial velocity is zero (u = 0)
b) acceleration a = -g
c) velocity and acceleration are in same direction.
d) Velocity attained after falling for a time t is v = gt (v = u + at)

Time taken to attain velocity v is $t = v/g$

e) Distance covered in a time t is $h = \frac{1}{2} gt^2$ ($s = ut + \frac{1}{2} at^2$)

Time taken to fall a distance h is $t = \sqrt{\frac{2h}{g}}$

f) Velocity attained after falling a distance h is $v = \sqrt{2gh}$ ($v^2 = u^2 + 2as$)

Distance covered when it attains velocity v is $h = \frac{v^2}{2g}$

g) Distance traveled in the n^{th} sec, is $s_n = g(n - \frac{1}{2})$

h) Ratio of height covered by a freely falling body in 1^{st}, 2^{nd}, 3^{rd} …n^{th} second is 1:3:5:7…..(2n-1)

i) Ratio of height covered by a freely falling body in 1, 2 , 3….n second is 1:4:9…..n^2

j) When there is no air resistance, equation of motion are independent of mass of the body.

k) If two particles of different masses are dropped simultaneously from the same height, they reach the ground at the same time with same velocity.

l) If a body under the influence of air resistance, as long as the velocity is increasing $a < g$ and acceleration decreases with time.

m) In the above case, the body after some time has no acceleration, and subsequently falls with a constant velocity called terminal velocity.

n) If two bodies are held one another separated by a distance s and released simultaneously, the distance of seperation between them remains same throughout their motion.

4) If a body falls freely from the top of a tower of height h and simultaneously another body is projected vertically upwards from the bottom of the tower, with a velocity u then they meet after time $\frac{h}{u}$

5) When a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.

6) If a body is projected vertically up from the top a tower of height ‘h’ with a velocity u and takes t seconds to reach the ground then net displacement $h = -ut + \frac{1}{2} gt^2$

7) If an object is dropped from a balloon rising up with a velocity u at a height h from the ground.

Equation of motion relative to earth is $h = -ut + \frac{1}{2} gt^2$

Equation of motion relative to balloon is $h = \frac{1}{2} gt^2$

Relative to earth body goes up and then falls.

Relative to the balloon it falls vertically downward.

8) A particle projected vertically up from the top of a tower takes time t_1 to reach the ground. Another particle thrown downwards with the same velocity from the top of the tower takes time t_2 to reach the ground. If the particle is dropped from the top of the tower, time taken is

$t = \sqrt{t_1 t_2}$

Height of the tower is $h = \frac{1}{2} g t_1 t_2$

Velocity of projection is $u = \frac{g}{2}(t_1 - t_2)$

In the first and second case body reaches the ground with the same velocity

9) A body is projected vertically upwards with a velocity u. It crosses a point in its journey at a height h twice, just after t_1 and t_2.

$h = ut_1 - \frac{1}{2} gt_1^2$

$h = ut_2 - \frac{1}{2} gt_2^2$

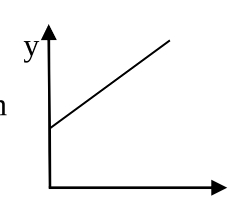
$u (t_2 - t_1) = \frac{1}{2} g(t_2^2 - t_1^2)$

Using $u = \frac{1}{2} g\left(t_2 + t_1\right)$

GRAPH

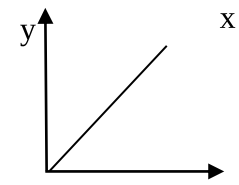
1) A linear equation between x and y represents a straight line, which is of the form $y = mx + c$

e.g. In $v = at + u$, v Vs t is a straight line



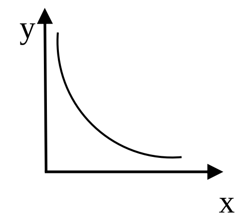
2) $y = mx$ represent a straight line passing through the origin

e.g In $s = vt$, s Vs t is a straight line through origin



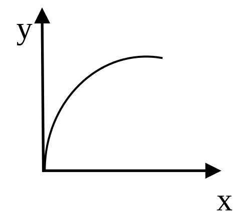
3) $x \propto \frac{1}{y}$ or xy = constant represents a rectangular hyperbola

e.g For isothermal process PV = constant. P Vs V is a rectangular hyperbola

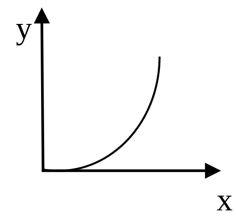


4) A quadratic equation in x and y represents a parabola in x-y graph.
e.g. $y = ax^2 + b$, $y^2 = ax$, $y = ax^2$
$s = ut + ½ at^2$, $v^2 = 2as$, $K = ½ mv^2$

$y^2 = ax$



$y = ax^2$



5) Slope of y-x graph = $\tan\theta = \frac{dy}{dx}$

6) Area under y-x graph = (dy) (dx)

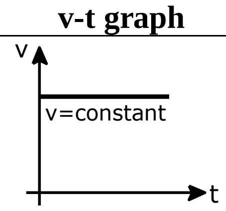
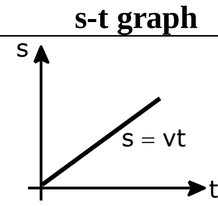
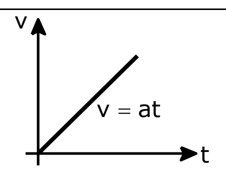
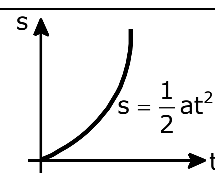
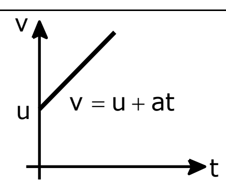
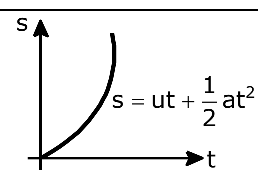
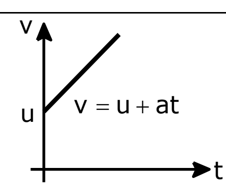
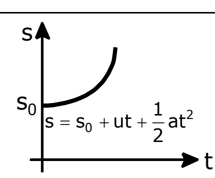
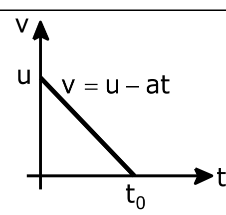
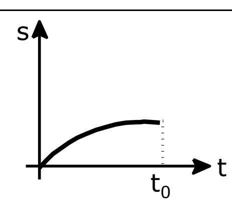
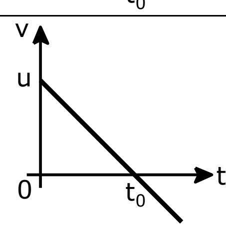
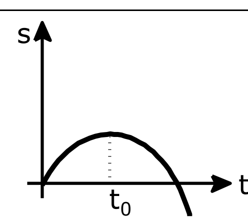
7) Slope of x-t graph gives the instantaneous velocity of the body.

8) For two particles having displacement-time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{v_1}{v_2} = \frac{\tan\theta_1}{\tan\theta_2}$

9) The slope of the velocity-time graph gives the acceleration of the body.
Greater the slope greater will be the acceleration

10) The area under the velocity time curve and time-axis gives the displacement of the body.

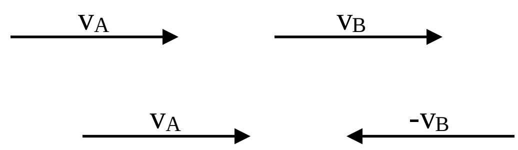
11) The area under a-t graph and time axis gives the change in velocity of the body during that time interval. (Area = a × (dt) = dv)

S.No	**Different cases**	**v-t graph**	**s-t graph**	**Important points**
1.	Uniform motion			(i) Slope of s-t graph = v = constant (ii) in s-t graph s = 0 at t = 0
2.	Uniformly accelerated motion with u = 0 and s =0 at t = 0			(i) u = 0, i.e., v = 0 at t = 0 (ii) a or slope v-t graph is constant (iii) u = 0, i.e., slope of s-t graph at t = 0, should be zero
3.	Uniformly accelerated motion with $u \neq 0$ but s = 0 at t = 0			(i) $u \neq 0$, i.e., v or slope of s-t graph at t = 0 is not zero (ii) v or slope of s-t graph gradually goes on increasing
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0			(i) v = u at t = 0 (ii) $s = s_0$ at t = 0
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s-t graph at t = 0 gives u. (ii) Slope of s-t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero
6.	Uniformly retarded and then accelerated in opposite direction			(i) At time $t = t_0$, v = 0 or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

RELATIVE VELOCITY

1) If the two bodies A and B are moving with velocities v_A and v_B in the same direction then, Relative velocity of A w.r.t B is $v_{AB} = v_A - v_B$

Relative velocity of B w.r.t A is $v_{BA} = v_B - v_A$



2) If the two bodies A and B are moving with velocities v_A and v_B in the opposite directions then, Relative velocity of A w.r.t B is $v_{AB} = v_A - (-v_B) = v_A + v_B$

Relative velocity of B w.r.t A is $v_{BA} = -(v_B + v_A)$

3) A motorboat covers a given distance in time t_1 moving downstream on a river. It covers the same distance in a time t_2 moving upstream. Then time it takes to cover the same distance in still water is $t = \frac{2t_1t_2}{t_1 + t_2}$

4) A girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator takes her up in a time t_2. The time taken by her to walk up on the moving escalator is $t = \frac{t_1t_2}{t_1 + t_2}$

5) Relative acceleration of A with respect to B is $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

Relative acceleration of B with respect to A is $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$

6) An elevator is accelerating upwards with an acceleration a. If a person inside the elevator throws a particle vertically up with a velocity u relative to the elevator, time of flight is $t = \frac{2u}{g + a}$

In the above case if elevator accelerates down, time of flight is $t = \frac{2u}{g - a}$