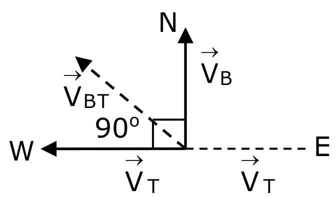
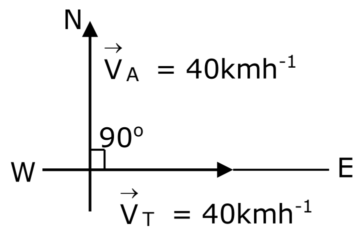
I PUC – CHAPTER 04
MOTION IN A PLANE

1. A bird of flying towards north with a velocity $40\,km\,h^{-1}$ and a train is moving with velocity $40\,km\,h^{-1}$ towards east. What is the velocity of the bird noted by a man in the train?
(a) $40\sqrt{2}$ km h^{-1} N-E (b) $40\sqrt{2}\,km\,h^{-1}$ S-E (c) $40\sqrt{2}\,km\,h^{-1}$ N-W (d) $40\sqrt{2}\,km\,h^{-1}$ S-W

(c)
To find the relative velocity of bird w.r.t train, superimpose velocity $-\vec{V}_T$ on both the objects. Now as a result of it, the train is at rest, while the bird possesses two velocities, $\vec{V}_B$ towards north and $-\vec{V}_T$ along west.



(a)



(b)

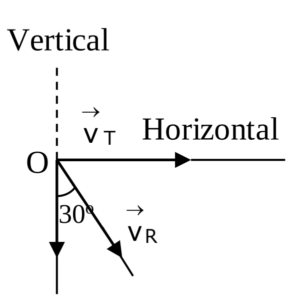
$|\vec{V}_{BT}| = \sqrt{|\vec{V}_B|^2 + |-\vec{V}_T|^2}$ [By formula, $\theta = 90^0$]= $\sqrt{40^2+40^2} = 40\sqrt{2}\,kmh^{-1}$ north-west.

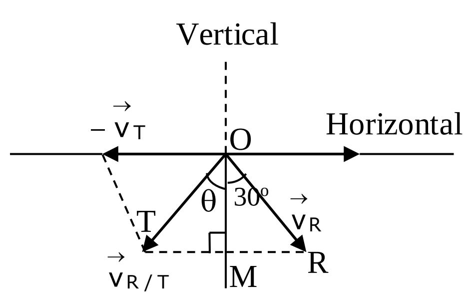
2. A swimmer wishes to cross a 500 m river flowing at $5\,kmh^{-1}$. His speed with respect to water is $3\,kmh^{-1}$. The shortest possible time to cross the river is
(a) 10 min (b) 20 min (c) 6 min (d) 7.5 min

(a)
d = 500 m = 0.5 km
Shortest time = $\dfrac{d}{v} = \dfrac{0.5}{3} = \dfrac{1}{6}h = 10\,\text{min}$

3. Rain, driven by the wind, falls on a railway compartment with a velocity of $20\,ms^{-1}$, at an angle of 30^0 to the vertical. The train moves, along the direction of wind flow, at a speed of $108\,km\,h^{-1}$. Determine the apparent velocity of rain for a person sitting in the train.
(a) $20\sqrt{7}\,ms^{-1}$ (b) $10\sqrt{7}\,ms^{-1}$ (c) $15\sqrt{7}\,ms^{-1}$ (d) $10\sqrt{7}\,kmh^{-1}$

(b)
Speed of train = $108 \times \dfrac{5}{18} = 30\,ms^{-1}$
Let $\vec{V}_R$ and $\vec{V}_T$ represent the respective velocities of rain and train.
Now, the relative velocity of rain w.r.t person (train) is given by $\vec{v}_{R,T} = \vec{v}_R - \vec{v}_T \Rightarrow \vec{v}_R + (-\vec{v}_T)$
Let $\overrightarrow{OR}$ and $\overrightarrow{RT}$ represent the vectors, respectively, in magnitude and direction.





$OT^2 = OR^2 + RT^2 + 2OR \cdot RT\cos 120^0 = 20^2 + 30^2 - 2\times 20\times 30\times \frac{1}{2} = 400 + 900 - 600 = 700 = \sqrt{700}\, ms^{-1}$

$= 10\sqrt{7}\, ms^{-1}$.

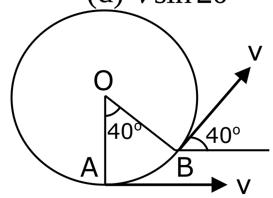
4. A particle is moving in a circle of radius $'r'$ centred at O with constant speed v. What is the change in velocity in moving from A to B ($\angle AOB = 40^0$)?

(a) $2v\sin 20^0$ (b) $4v\sin 40^0$ (c) $2v\sin 40^0$ (d) $v\sin 20^0$

(a)

Change in velocity = $\vec{v}_f - \vec{v}_i$

Its magnitude is = $\sqrt{v^2 + c^2 - 2vv\cos 40^0}$ = $2v\sin 20^0$



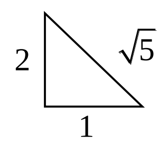
5. A particle is projected with a velocity v so that its range on a horizontal plane is twice the greatest height attained. If g is acceleration due to gravity, then its range is

(a) $\frac{4v^2}{5g}$ (b) $\frac{4g}{5v^2}$ (c) $\frac{4v^3}{5g^2}$ (d) $\frac{4v}{5g^2}$

(a)

Since R = 2H

$\frac{v^2\sin 2\theta}{g} = 2\times\frac{v^2\sin^2\theta}{2g}$ or $2\sin\theta\cos\theta = \sin^2\theta$ or $\tan\theta = 2$.



$R = \frac{v^2\sin 2\theta}{g} = \frac{v^2 2\sin\theta\cos\theta}{g} = \frac{2v^2}{g}\times\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$

6. A shot is fired from a point at a distance of 200 m from the foot of a tower 100 m high so that it just passes over in horizontally. The direction of shot with horizontal is

(a) 30^0 (b) 45^0 (c) 60^0 (d) 70^0

(b)

H =100 m, R = $2\times 200 = 400\,m$

$\tan\theta = \frac{4H}{R} \Rightarrow \tan\theta = \frac{4\times 100}{400} = 1 \Rightarrow \theta = 45^0 \left[\because \frac{H}{R} = \frac{\tan\theta}{4}\right]$.

7. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

(a) Yes, 60^0 (b) Yes, 30^0 (c) No (d) Yes, 45^0

(a)

For the person to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e., $v_0 \cos\theta = \frac{v_0}{2}$ or $\cos\theta = \frac{1}{2}$ or $\theta = 60^0$.

8. A body has an initial velocity of $3ms^{-1}$ and has an acceleration of $1ms^{-2}$ normal to the direction of the initial velocity. Then its velocity 4s after the start is

(a) $7ms^{-1}$ along the direction of initial velocity

(b) $7ms^{-1}$ along the normal to the direction of initial velocity.

(c) $7ms^{-1}$ midway between the two directions.

(d) $5ms^{-1}$ at an angle $\tan^{-1}(4/3)$ with the direction of initial velocity.

(d)

Let $u_x = 3ms^{-1}$, $a_x = 0$

$v_y = u_y + a_y t = 0 + 1 \times 4 = 4ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = 5m/s$

Angle made by the resultant velocity w.r.t direction of initial velocity, i.e., x-axis is

$\beta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{4}{3}$.

9. The equation of motion of a projectile is y = 12x - $\frac{3}{4}x^2$. The horizontal component of velocity is $3ms^{-1}$. What is the range of the projectile?

(a) 18m (b) 16m (c) 12m (d) 21.6m

(b)

Given $y = 12x - \frac{3}{4}x^2$, $u_x = 3ms^{-1}$

$y = ax - bx^2$

$R = \frac{a}{b} = \frac{12}{3/4} = 16m$

10. A ball is thrown upwards at an angle of 60° to the horizontal. It falls on the ground at a distance of 90 m. If the ball is thrown with the same initial velocity at an angle of 30°, it will fall on the ground at a distance of

(a) 120 m (b) 90 m (c) 60 m (d) 30 m

(b)

For the angles of projection θ and (90-θ), range is same.

11. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of $150ms^{-1}$. Then the time after which its inclination with the horizontal is 45° is ($g = 10\ ms^{-2}$)

(a) 15($\sqrt{3}$ -1)s (b) 15($\sqrt{3}$ +1)s (c) 7.5($\sqrt{3}$ -1)s (d) 7.5($\sqrt{3}$ +1)s

(c)
At the two points of the trajectory during projectile motion, the horizontal component of the velocity is same. Then u $\cos\theta_1$ = v $\cos\theta_2$

$$150\times\frac{1}{2}=v\times\frac{1}{\sqrt{2}} \text{ or } v=\frac{150}{\sqrt{2}}ms^{-1}$$

Initially: $u_y = u\sin 60^0 = \frac{150\sqrt{3}}{2}ms^{-1}$

Finally: $v_y = v\sin 45^0 = \frac{150}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = \frac{150}{2}ms^{-1}$

But $v_y = u_y + a_y t$ or $\frac{150}{2}=\frac{150\sqrt{3}}{2}-10t$

$10t=\frac{150}{2}(\sqrt{3}-1)$ or $t=7.5(\sqrt{3}-1)$.

12. A cannon on a level ground is aimed at an angle α above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff at distance R away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(a) R sin α - $\frac{1}{2}\frac{gR^2}{v_0^2\sin^2\alpha}$ (b) R cos α - $\frac{1}{2}\frac{gR^2}{v_0^2\cos^2\alpha}$

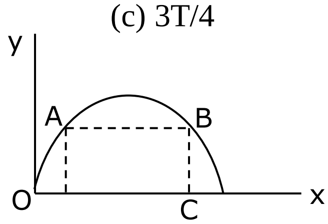
(c) R tan α - $\frac{1}{2}\frac{gR^2}{v_0^2\cos^2\alpha}$ (d) R tan α - $\frac{1}{2}\frac{gR^2}{v_0^2\sin^2\alpha}$

(c)
The time taken to move a horizontal distance R is t = R/(v_0 cos α). Therefore, the vertical distance moved in this time is given by h = u_0t - $\frac{1}{2}gt^2$

$$= v_0\sin\alpha \times \frac{R}{v_0\cos\alpha} - \frac{1}{2}g\left(\frac{R}{v_0\cos\alpha}\right)^2 = R\tan\alpha - \frac{gR^2}{2v_0^2\cos^2\alpha}$$

13. There are two values of time for which a projectile is at the same height. The sum of these two times is equal to (T = Time of flight of the projectile)

(a) 3T/2 (b) 4T/3 (c) 3T/4 (d) T



(d)

$t_{OA}=t_{BC}$

To find $t_{OA}+t_{OB}$

$t_{OA}+t_{OB}=t_{BC}+(t_{OB})=T$.

14. The trajectory of a projectile in a vertical plane is $y=ax-bx^2$, where 'a' and 'b' are constants and 'x' and 'y' are respectively, horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(a) $\frac{b^2}{2a},\tan^{-1}(b)$ (b) $\frac{a^2}{b},\tan^{-1}(2b)$ (c) $\frac{a^2}{4b},\tan^{-1}(a)$ (d) $\frac{2a^2}{b},\tan^{-1}(a)$

(c)

$y = ax - bx^2$, for height or y to be maximum, $\frac{dy}{dx} = 0$ or $a - 2bx = 0$ or $x = \frac{a}{2b}$

(i) $y_{\max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$

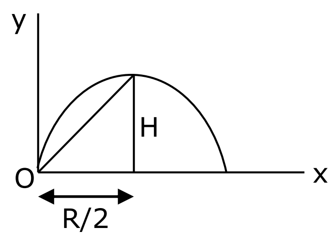
(ii) $\left(\frac{dy}{dx}\right)_{x=0} = a = \tan\theta_0$, where θ_0 = angle of projection

$\theta_0 = \tan^{-1}(a)$

15. A particle is projected from the ground with an initial speed of 'v' at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(a) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ (b) $\frac{v}{2}\sqrt{1+\cos^2\theta}$ (c) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ (d) $v\cos\theta$

(c)



Average velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{\sqrt{H^2 + R^2/4}}{T/2}$.

Replacing H, R and T, We get $v_{av} = \frac{y}{2}\sqrt{1+3\cos^2\theta}$

16. A body of mass m is projected horizontally with a velocity 'v' from the top of a tower of height 'h' and it reaches the ground at a distance 'x' from the foot of the tower. If a second body of mass 2m is projected horizontally from the top of a tower of height 2h, it reaches the ground at a distance '2x' from the foot of the tower. The horizontal velocity of the second body is

(a) v (b) 2v (c) $\sqrt{2}$ v (d) v/2

(c)

x = v T and T = $\sqrt{\frac{2h}{g}}$

$x = v\sqrt{\frac{2h}{g}}$, $2x = v'\sqrt{\frac{2(2h)}{g}}$ solve to get $v' = \sqrt{2}v$

17. The horizontal range and maximum height attained by a projectile are R and H respectively. If a constant horizontal acceleration $a = g/4$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(a) $(R+H), \frac{H}{2}$ (b) $\left(R+\frac{H}{2}\right), 2H$ (c) $(R+2H), H$ (d) $(R+H), H$

(d)

$H = \frac{u^2\sin^2\theta}{2g}$, $R = \frac{u^2\sin 2\theta}{g}$

Maximum height will be the same because acceleration a = g/4 is along horizontal.

$R' = u\cos\theta T + \frac{1}{2}aT^2 = R + \frac{1}{2}\frac{g}{4}\left(\frac{2u\sin\theta}{g}\right)^2 = R + H$

18. A body is moving in a circle with a speed of $1ms^{-1}$. This speed increases at a constant rate of $2\,ms^{-1}$ every second. Assume that the radius of the circle described is 25 m. The total acceleration of the body after 2s is

(a) $2\,ms^{-2}$ (b) $25\,ms^{-2}$ (c) $\sqrt{5}\,ms^{-2}$ (d) $\sqrt{7}\,ms^{-2}$

(c)

$a_t = 2ms^{-2}$, $v = u + a_t t = 1 + 2\times 2 = 5ms^{-1}$ $\quad a_c = \frac{v^2}{r} = \frac{5^2}{25} = 1ms^{-2}$

Net acceleration= $\sqrt{a_c^2 + a_t^2} = \sqrt{1^2 + 2^2} = \sqrt{5}\,ms^{-2}$.

19. A boat is moving with a velocity $3\hat{i} + 4\hat{j}$ with respect to ground. The water in the river is moving with a velocity $-3\hat{i} - 4\hat{j}$ with respect to ground. The relative velocity of the boat with respect to water is

(a) $8\,\hat{j}$ (b) $-6\hat{i} - 8\hat{j}$ (c) $6\hat{i} + 8\hat{j}$ (d) $5\sqrt{2}$.

(c)

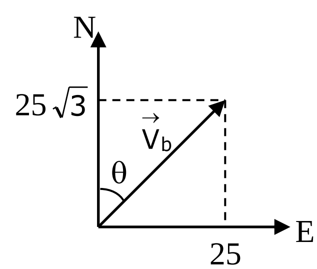
Relative velocity of boat with respect to water is $\vec{v}_b - \vec{v}_w = 3\hat{i} + 4\hat{j} - (-3\hat{i} - 4\hat{j}) = 6\hat{i} + 8\hat{j}$

20. A car is moving towards east with a speed of $25\,km\,h^{-1}$. To the driver of the car, a bus appears to move towards north with a speed $25\sqrt{3}\,km\,h^{-1}$. What is the actual velocity of the bus?

(a) $50\,km\,h^{-1}$, 30^0 E of N (b) $50\,km\,h^{-1}$, 30^0 N of E

(c) $25\,km\,h^{-1}$, 30^0 E of N (d) $25\,km\,h^{-1}$, 30^0 N of E

(a)



$\vec{v}_c = 25\hat{i}$, $\vec{v}_{b/c} = 25\sqrt{3}\hat{j}$

$\vec{V}_{b/c} = \vec{V}_b - \vec{V}_c \Rightarrow \vec{V}_b = \vec{V}_{b/c} + \vec{V}_c$

$\Rightarrow \vec{v}_b = 25\hat{i} + 25\sqrt{3}\hat{j}$

$|\vec{v}_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50\,km\,h^{-1}$

$\tan\theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^0$.

21. During a projectile motion, if the maximum height equals the horizontal range, then the angle of projection with the horizontal is

(a) $\tan^{-1}(1)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}(3)$ (d) $\tan^{-1}(4)$

(d)

We find that H = R or $\frac{u^2\sin^2\theta}{2g} = \frac{u^2 2\sin\theta\cos\theta}{g}$ or $\tan\theta = 4 \text{ or } \theta = \tan^{-1}(4)$.

22. A projectile has a time of flight T and range R. If the time of flight is doubled, keeping the angle of projection same, what happens to the range?

(a) R/4 (b) R/2 (c) 2R (d) 4R

(d)

$$\frac{R}{T^2}=\frac{u^2\sin 2\theta/g}{4u^2\sin^2\theta/g^2}=\frac{g}{2}\cot\theta \qquad \text{i.e., } gT^2=2R\tan\theta$$

If T is doubled, then R becomes 4 times.

23. At a height 0.4m from the ground, the velocity of a projectile in vector form is $\vec{v}=(6\hat{i}+2\hat{j})ms^{-1}$. The angle of projection is (g = 10 m/s^2)

(a) 45^0 (b) 60^0 (c) 30^0 (d) $\tan^{-1}\left(\frac{3}{4}\right)$

(c)

$v^2=u^2-2gh$ or $u^2=v^2+2gh$ or $u_x^2+u_y^2=v_x^2+v_y^2+2gh$, $u_x=v_x$

So, $u_y^2=v_y^2+2gh$ or $u_y^2=(2)+2\times10\times0.4=12$

$u_y=\sqrt{12}=2\sqrt{3}\,ms^{-1}$, $u_x=v_x=6ms^{-1}$

$$\tan\theta=\frac{u_y}{u_x}=\frac{2\sqrt{3}}{6}=\frac{1}{\sqrt{3}}\Rightarrow\theta=30^0$$

24. A hose lying on the ground shoots a stream of water upward at an angle of 60^0 to the horizontal with the velocity of $16\,ms^{-1}$. The height at which the water strikes the wall 8m away is (g 10 m/s^2)

(a) 8.9 m (b) 10.9 m (c) 12.9 m (d) 6.9 m

(a)

$u_x=u\cos\theta=16\cos 60^0=8ms^{-1}$

Time taken to reach the wall = 8/8 = 1s

Now $u_y=u\sin\theta=16\sin 60^0=8\sqrt{3}\,ms^{-1}$

$$h=(u\sin\theta)\text{t}+\frac{1}{2}gt^2=8\sqrt{3}\times1-\frac{1}{2}\times10\times1=13.9-5=8.9\,m$$

25. Two paper screens A and B are separated by 150 m. A bullet pierces A and B. The hole in B is 15 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is ($g=10\,ms^{-2}$)

(a) $100\sqrt{3}\,ms^{-1}$ (b) $200\sqrt{3}\,ms^{-1}$ (c) $300\sqrt{3}\,ms^{-1}$ (d) $500\sqrt{3}\,ms^{-1}$

(d)

Range =50 = ut and $h=\frac{15}{100}=\frac{1}{2}\times gt^2$

$$\text{or}\, t^2=\frac{2\times15}{100\times g}=\frac{30}{1000}\,\text{or}\, t=\frac{\sqrt{3}}{10}$$

$$u=\frac{v}{t}=\frac{150}{t}=\frac{150\times10}{\sqrt{3}}=500\sqrt{3}\,ms^{-1}$$

26. A ball is thrown at different angles with the same speed u from the same point and it has the same range in both the cases. If y_1 and y_2 are the heights attained in the cases, then y_1+y_2 is equal to

(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$ (c) $\frac{u^2}{2g}$ (d) $\frac{u^2}{4g}$

(c)
Range for both the projectiles is same if $\theta_1 + \theta_2 = 90^o$

$$y_1 = \frac{u^2 \sin^2\theta}{2g},\ y_2 = \frac{u^2 \sin^2(90^0 - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g} \Rightarrow y_1 + y_2 = \frac{u^2}{2g}$$

27. A vector A is along the positive z-axis and its vector product with another vector B is zero, then vector B could be

(a) $\hat{i} + \hat{j}$ (b) $4\hat{i}$ (c) $\hat{j} + \hat{k}$ (d) $-7\hat{k}$

(d)
The vector product of two non-zero vectors is zero if they are in the same direction. Hence, vector B must be parallel to vector A, i.e. along ± z-axis.

28. A golfer standing on level ground hits a ball with a velocity of $u = 52\,ms^{-1}$ at an angle α above the horizontal. If $\tan\alpha = 5/12$, then time for which the ball is at least 15m above the ground will be (take $g = 10\,ms^{-2}$)

(a) 1 s (b) 2 s (c) 3 s (d) 4 s

(b)
When $\tan\alpha = 5/12$ then $\sin\alpha = 5/13$
Let at any time t, the ball is at height of 15 m.

$$S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 15 = u \sin\alpha t - \frac{1}{2} g t^2 \Rightarrow 15 = 52 \times \frac{5}{13} t - \frac{1}{2} 10 t^2$$

$$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0 \Rightarrow t = 1s,\ t = 3s.$$

Required time is 3 – 1 = 2s.

29. The speed of a projectile at its maximum height is $\sqrt{3}/2$ times its initial speed. If the range of the projectile is P times the maximum height attained by it, P is equal to

(a) 4/3 (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) 3/4

(c)

Given $\frac{\sqrt{3}u}{2} = u\cos\theta$ = speed at maximum height

or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = 30^0$ …. (1)

Given that $PH_{max} = R$ ….. (2)

We know $H_{max} = \frac{R\tan\theta}{4}$

$$P = \frac{R}{H_{max}} = \frac{4}{\tan\theta} = \frac{4}{\tan 30^0} = 4\sqrt{3}$$

30. A projectile is given an initial velocity of $\hat{i}+2\hat{j}$. The Cartesian equation of its path is ($g = 10\,ms^{-2}$).

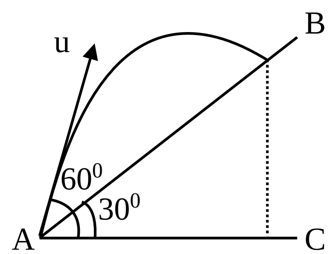
(a) $y = 2x - 5x^2$ (b) $y = x - 5x^2$ (c) $4y = 2x - 5x^2$ (d) $y = 2x - 25x^2$.

(a)

$$\tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1}$$

The desired equation is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ $= x \times 2 \frac{10x^2}{2(\sqrt{2^2+1^2})^2\left(\frac{1}{\sqrt{5}}\right)^2}$ or $y = 2x - 5x^2$.

31. In figure, the time taken by the projectile to reach from A to B is t. Then the distance AB is equal to



(a) $\frac{ut}{\sqrt{3}}$ (b) $\frac{\sqrt{3}ut}{2}$

(c) $\sqrt{3}\,ut$ (d) $2\,ut$

(a)

Horizontal component of velocity, $u_H = u\cos 60^0 = \frac{u}{2}$

$AC = u_H \times t = \frac{ut}{2}$ and AB $= AC\sec 30^0 = \left(\frac{ut}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = \frac{ut}{\sqrt{3}}$.

32. The angular velocity of a particle moving in a circle of radius 50 cm is increased in 5 min from 100 revolutions per minute to 400 revolutions per minute. Find the tangential acceleration of the particle.

(a) $60\,ms^{-2}$ (b) $\frac{\pi}{30}ms^{-2}$ (c) $\frac{\pi}{15}ms^{-2}$ (d) $\frac{\pi}{60}ms^{-2}$

(d)

$\alpha = (\omega_2 - \omega_1)/t = (400-100)/5 = 60\ rev\,\text{min}^{-2}$ $= \frac{60\times 2\pi}{(60)^2} = \frac{2\pi}{60}\text{rad}\,\text{s}^{-2}$

$a_t = \alpha r = \frac{2\pi}{60}\times\frac{50}{100} = \frac{\pi}{60}ms^{-2}$.

33. A is a vector which when added to the resultant of vectors $(2\hat{i} - 3\hat{j} + 4\hat{k})$ and $(\hat{i} + 5\hat{j} + 2\hat{k})$ yields a unit vector along the y-axis. Then vector A is

(a) $-3\hat{i} - \hat{j} - 6\hat{k}$ (b) $3\hat{i} + \hat{j} - 6\hat{k}$ (c) $3\hat{i} - \hat{j} + 6\hat{k}$ (d) $3\hat{i} + \hat{j} + 6\hat{k}$

(a)

$\vec{A} + (2\hat{i} - 3\hat{j} + 4\hat{k}) + (\hat{i} + 5\hat{j} + 2\hat{k}) = 1\hat{j}$ Or $\vec{A} = -3\hat{i} - \hat{j} - 6\hat{k}$

34. The point from where a ball is projected is taken as the origin of the coordinate axes. The x and y components of its displacement are given by $x = 6t$ and $y = 8t - 5t^2$. What is the velocity of projection?
(a) 6 ms^{-1} (b) 8 ms^{-1} (c) 10 ms^{-1} (d) 14 ms^{-1}

(c)
$v_x = \frac{dx}{dt} = 6$ m/s and $v_y = \frac{dy}{dt} = 8 - 10t$.
For the point of projection, t =0. $\therefore$ $v_y = 8 - 10 \times 0 = 8$ m/s.
$\therefore$ Velocity of projection $v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10\ ms^{-1}$

35. The angle between two vectors A and B is θ. Vector R is the resultant of the two vectors. If R makes an angle $\frac{\theta}{2}$ with A, then
(a) A = 2B (b) $A = \frac{B}{2}$ (c) A = B (d) AB = 1

(c)
The angle α which the resultant R makes with A is given by $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$
Given $\alpha = \frac{\theta}{2}$. Hence $\tan\left(\frac{\theta}{2}\right) = \frac{B\sin\theta}{A + B\cos\theta}$
or $\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{2B\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{A + B\cos\theta}$
Which gives $A + B\cos\theta = 2B\cos^2\left(\frac{\theta}{2}\right)$
or $A + B\left[2\cos^2\left(\frac{\theta}{2}\right) - 1\right] = 2B\cos^2\left(\frac{\theta}{2}\right)$
Which gives A = B

36. A body of mass 2 kg has an initial velocity of 3 m/s along x-axis and it is subjected to a force of 4N in y-direction. The distance of the body from origin after 4 s will be: (the body was subjected to force at the origin at t = 0)
(a) 12 m (b) 28 m (c) 20 m (d) 48 m

(c)
$u_x = 3\ ms^{-1}$, $a_x = 0$, $u_y = 0$, $a_y = \frac{F}{m} = \frac{4}{2} = 2 m/s^2$ and t = 4 s
If s_x and s_y be the displacement along x-axis and y-axis respectively then
$s_x = u_x t + \frac{1}{2} a_x t^2 = 3 \times 4 + \frac{1}{2} \times 0 \times (d)^2 = 12$ m
and $s_y = u_y t + \frac{1}{2} a_y t^2 = 0 \times 4 + \frac{1}{2} \times 2 \times (4)^2 = 16$m
$\therefore$ Displacement from the origin $s = \sqrt{s_x^2 + s_y^2} = \sqrt{(12)^2 + (16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$m

37. The initial velocity of a particle is u = $(4\hat{i} + 3\hat{j})$ m/s. It is moving with uniform acceleration a = $(0.4\hat{i} + 0.3\hat{j})$ m/s^2. The magnitude of its velocity after 10s is

(a) 3 m/s (b) 3 m/s (c) 5 m/s (d) 10 m/s

(d)

$\vec{v} = \vec{u} + \vec{a}t = (4\hat{i} + 3\hat{j}) + (0.4\hat{i} + 0.3\hat{j})$ x 10 = $(8\hat{i} + 6\hat{j})$ m/s

The x and y components of v are v_x = 8 m/s and v_y = 6 m/s.

The magnitude of v is v = $\sqrt{v_x^2 + v_y^2} = \sqrt{(8)^2 + (6)^2}$ = 10 m/s

38. If a_r and a_t represent radial and tangential accelerations, the motion of a particle will be uniformly circular if

(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ but $a_t \neq 0$ (c) $a_r \neq 0$ but $a_t = 0$ (d) $a_r \neq 0$ and $a_t \neq 0$

(c)

For uniform circular motion, acceleration is directed towards the centre known as centripetal or radial acceleration. The acceleration in the direction of velocity is zero. i.e, tangential acceleration is zero.

39. From the top of a tower 19.6 m high, a ball is thrown horizontally. If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, then the initial velocity of the ball is (g = 9.8 m/s^2)

(a) 9.8 ms^{-1} (b) 4.9 ms^{-1} (c) 14.7 ms^{-1} (d) 2.8 ms^{-1}

(a)

Since angle with the horizontal is 45°, therefore vertical height = range

Ie, H = R.

Time for fall $t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2\,s$ (using s= ut +1/2 at^2)

For horizontal distance H = vt

ie 19.6 = v × 2 or v = 9.8 ms^{-1}

40. Two tall buildings are 30 m apart. The speed with which a ball must be thrown horizontally from a window 150 m above the ground from one building so that it enters a window 27.5 m from the ground in the other building is (g 9.8 m/s^2)

(a) 2 ms^{-1} (b) 6 ms^{-1} (c) 4 ms^{-1} (d) 8 ms^{-1}

(b)

Fall in the height of the ball

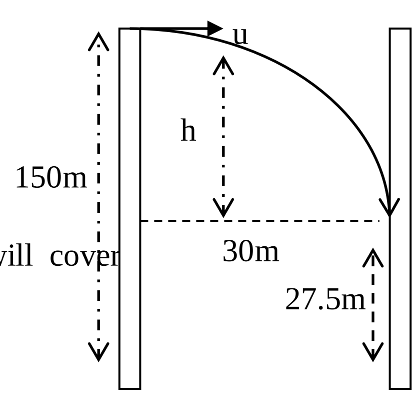
h = 150 – 27.5 = 122.5 m

∴ Time taken for the fall

t = $\sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 122.5}{9.8}}$ = 5 s

In 5s, ball falls by 122.5m and in the same time it will cover horizontal distance of 30m.

Hence u = $\frac{R}{t} = \frac{30}{5}$ = 6ms^{-1}



41. Given: $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{B} = -\hat{i} - \hat{j} + \hat{k}$. The unit vector of $\vec{A} - \vec{B}$ is

(a) $\frac{3\hat{i} + \hat{k}}{\sqrt{10}}$ (b) $\frac{3\hat{i}}{\sqrt{10}}$ (c) $\frac{\hat{k}}{\sqrt{10}}$ (d) $\frac{-3\hat{i} - \hat{k}}{\sqrt{10}}$

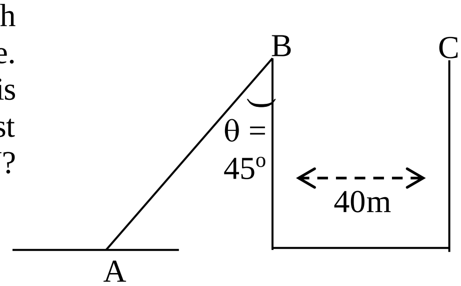
(a)

$\vec{A} - \vec{B} = 3\hat{i} + \hat{k}$

$|\vec{A} - \vec{B}| = \sqrt{3^2 + 1^2} = \sqrt{10}$

Required unit vector is $\frac{3\hat{i} + \hat{k}}{\sqrt{10}}$ $[\hat{A} = \frac{\vec{A}}{A}]$

42. A body is projected up a smooth inclined plane with velocity V from the point A as shown in the figure. The angle of inclination is 45° and the top is connected to a well of diameter 40m. If the body just manages to cross the well, what is the value of V? Length of inclined plane is $20\sqrt{2}$ m.



(a) 40 ms^{-1} (b) $40\sqrt{2}$ ms^{-1}
(c) 20 ms^{-1} (d) $20\sqrt{2}$ ms^{-1}

(c)

Angle of projection from B is 45°. As the body is able to cross the well of diameter 40 m, hence R = $\frac{v^2}{g}$

or v = $\sqrt{gR}$ or v = $\sqrt{10 \times 40}$ = 20 ms^{-1}

43. The friction of the air causes a vertical retardation equal to 10% of the acceleration due to gravity (Take g = 10 ms^{-2}). The maximum height will be decreased by

(a) 8% (b) 9% (c) 10% (d) 11%

(b)

Retardation due to friction or air = $\frac{g}{10}$.

In upward motion: Total retardation = g + $\frac{g}{10} = \frac{11\,g}{10}$

$\therefore H_m = \frac{u^2 \sin^2\theta}{2g}$

$H'_m = \frac{u^2 \sin^2\theta}{2 \times \frac{11\,g}{10}} = \frac{10}{11} \times \frac{u^2 \sin^2\theta}{2\,g} = \frac{10}{11} H_m$

$\therefore$ % decrease in $H_m = \frac{H_m - H'_m}{H_m} \times 100 = \left(1 - \frac{10}{11}\right) \times 100 = 9\%$

44. A projectile has a maximum range of 16 km. At the highest point of its motion, it explodes into two equal masses. One mass drops vertically downwards. The horizontal distance covered by the other mass from the time of explosion is

(a) 8 km (b) 16 km (c) 24 km (d) 32 km

(b)

Maximum range $R_m = \frac{u^2}{g} = 16$ km.

Range is maximum when $\theta = 45^o$.

Initial momentum at the highest point = mu cos 45^o.

After explosion, the projectile breaks into two equal masses. As one mass drops vertically downwards, hence its initial momentum after explosion is zero.

Hence, if v be the velocity of second mass, then according to law of conservation of momentum,

$mu \cos 45^o = \frac{m}{2}.v$

$\therefore \quad v = 2u \cos 45^o$

Horizontal distance covered from the time of explosion = horizontal velocity after explosion x time of descent

$= v \times \frac{T}{2} = v \times \frac{1}{2} \times \frac{2u \sin 45^o}{g} = 2u \cos 45^o \times \frac{u \sin 45^o}{g} = \frac{u^2}{g} \times \sin 90^o = \frac{u^2}{g} = 16$ km

45. A stuntman plans to run across a roof top and then horizontally off it to land on the roof of next building. The roof of the next building is 4.9 m below the first one and 6.2 m away from it. What should be his minimum roof top speed in m/s, so that he can successfully make the jump? ($g = 9.8$ m/s^2)

(a) 3.1 (b) 4. 0 (c) 4.9 (d) 6.2

(d)

Time of descent $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 4.9}{9.8}} = 1s.$

Horizontal distance covered, $x = u\,t$.

Or $\quad u = \frac{x}{t} = \frac{6.2}{1} = 6.2 m/s.$

46. Two equal forces act at a point. The square of their resultant is three times their product. The angle between them is

(a) 0° (b) 30° (c) 60° (d) 90°

(c)

Given: Q = P ($\because$ both the forces are equal). $R^2 = 3PQ = 3P(P) = 3P^2$.

W.k.t $R^2 = P^2 + Q^2 + 2PQ\cos(\theta)$.

ie, $3P^2 = P^2 + P^2 + 2P(P)\cos(\theta) = 2P^2 + 2P^2\cos(\theta)$.

or $\quad \theta = \cos^{-1}(0.5) = 60^0$

47. A man walks 4m towards East and the 3m towards North and there he fixes a pole 12m high. The distance between the starting point and tip of the pole in space is

(a) 7m (b) 11 m (c) 13m (d) 19 m

(c)

$\vec{s} = 4\hat{i} + 3\hat{j} + 12\hat{k}$

$|\vec{s}| = s = \sqrt{4^2 + 3^2 + 12^2}\,m = \sqrt{169}\,m = 13m$

48. Vector C is the sum of two vectors A and B vector D is the cross product of vectors A and B. What is the angle between vectors C and D?
(a) zero (b) 60^o (c) 90^o (d) 180^o

(c)
Vector C lies in the plane containing vectors A and B, and vector D is perpendicular to both A and B. Hence D must be perpendicular to C.

49. A body is projected at time t = 0 from a certain point on a planet's surface with a certain velocity at a certain angle w.r.to surface (assumed horizontal). The horizontal and vertical displacements x and y (in m) respectively vary with time t (in second) as $x = 10\sqrt{3}\ t$ and $y = 10t - t^2$
What is the magnitude and direction of the velocity with which the body is projected?
(a) 20 m/s at an angle of 30^o with the horizontal
(b) 20 m/s at an angle of 60^o with the horizontal
(c) 10 m/s at an angle of 30^o with the horizontal
(d) 10 m/s at an angle of 60^o with the horizontal

(a)
$x = (u \cos\theta)\, t$ and $y = (u \sin\theta)\, t - \frac{1}{2} gt^2$
Comparing $x = 10\sqrt{3}$ we get $u\cos(\theta) = 10\sqrt{3}$
Comparing $y = 10t - t^2$, we have, $u \sin\theta = 10$
Squaring and adding above two equations, we get
$$u^2\left(\sin^2\theta + \cos^2\theta\right) = 10^2 + \left(10\sqrt{3}\right)^2 = 400$$
Or u = 20 m/s and $\tan\theta = \frac{1}{\sqrt{3}}$ which gives $\theta = 30^o$.

50. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/s at an angle of elevation of 30^o. The ratio of the total time taken by the ball to hit the ground to its time of flight (time taken to come back to the same elevation) is (Take $g = 10\ m/s^2$)
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 1.5 : 1

(a)
The time of flight $t_f = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times 0.5}{10} = 2$ s.
The initial downward velocity at same level during descending = 20 sin30^o = 10 m/s.
The time taken to fall through a height of 40 m with velocity 10 m/s is given by
$40 = 10 \times t + \frac{1}{2} \times 10 \times t^2$
or $t^2 + 2t - 8 = 0$
which gives t = 2s. Hence the total time to hit the ground $t_f + t = 2 + 2 = 4$s.