1PUC – CHAPTER 08
GRAVITATION

1. The distance of two planets from the sun are 10^{13} m and 10^{12} m respectively. The ratio of time periods of these two planets is

a) $\frac{1}{\sqrt{10}}$ b) 100 c) $10\sqrt{10}$ d) $\sqrt{10}$

(c)

$T^2 \propto R^3$ (Kepler's law)

$$\frac{T_1^2}{T_2^2} = \left(\frac{10^{13}}{10^{12}}\right)^3 \Rightarrow \frac{T_1}{T_2} = 10\sqrt{10}$$

2. The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major- axis of the orbit drawn from the sun is

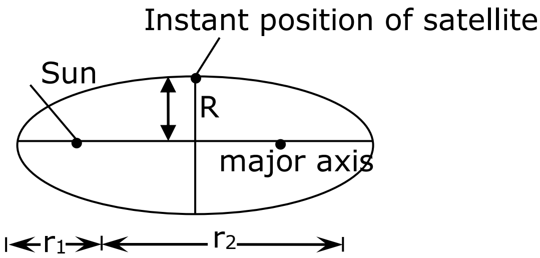
a) $\frac{r_1 + r_2}{4}$ b) $\frac{r_1 + r_2}{r_1 - r_2}$ c) $\frac{2r_1r_2}{r_1 + r_2}$ d) $\frac{r_1 + r_2}{3}$

(c)

Applying the properties of ellipse, we have

$$\frac{2}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2}$$

$$R = \frac{2r_1r_2}{r_1 + r_2}$$



3. If the gravitational force between two objects were proportional to 1/R (and not as $1/R^2$) where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to

a) $1/R^2$ b) R^0 c) R^1 d) 1/R

(b)

$$F = \frac{k}{R} = \frac{Mv^2}{R}.$$

Hence $v \propto R^0$

4. For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be

a) 11 km/s b) $11\sqrt{3}$ km/s c) $\frac{11}{\sqrt{3}}$ km/s d) 33 km/s

(a)

Since, escape velocity $\left(v_e = \sqrt{2gR_e}\right)$ is independent of angle of projection, it will not change.

5. A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct?

a) T is conserved
b) V is always positive
c) E is always negative
d) L is conserved but direction of vector L changes continuously

(c)

In circular or elliptical orbital motion, torque is always acting parallel to displacement or velocity. So, angular momentum is conserved. In attractive field, potential energy is negative. Kinetic energy changes as velocity increase when distance is less. So, option (c) is correct.

6. A satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy?

a) $(3/4)\ mv^2$ b) $(1/2)\ mv^2$ c) mv^2 d) $-(1/2)\ mv^2$

(d)

Total energy = -KE = $\frac{PE}{2}$

K.E = $\frac{1}{2}mv^2$

7. A seconds pendulum is mounted in a rocket. Its period of oscillation decreases when the rocket
a) comes down with uniform acceleration
b) moves round the earth in a geostationary orbit
c) moves up with a uniform velocity
d) moves up with a uniform acceleration
(d)
$T = 2\pi\sqrt{l/g}$. When the rocket accelerates upwards g increases to (g + a).

8. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite?
a) $(R^2 g/\omega^2)^{1/3}$ b) $(Rg/\omega^2)^{1/3}$ c) $(R^2\omega^2/g^2)^{1/3}$ d) $(R^2 g/\omega)^{1/3}$
(a)

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}}$$

$$r^3 = \frac{gR^2}{\omega^2}$$

$$r = \left(\frac{gR^2}{\omega^2}\right)^{1/3}$$

9. A satellite in force free space sweeps stationary interplanetary dust at a rate dM/dt = αv where M is the mass and v is the velocity of the satellite and α is a constant. What is the deceleration of the satellite?
a) $-\alpha v^2$ b) $-\alpha v^2/2M$ c) $-\alpha v^2/M$ d) $-2\alpha v^2/M$
(c)

$$F = \left(\frac{dM}{dt}\right)v = \alpha v^2 \qquad \left(\because \frac{dM}{dt} = \alpha v\right)$$

$$\therefore \text{Retardation} = \frac{-F}{M} = -\frac{\alpha v^2}{M}$$

10. The escape velocity from the surface of the earth is v_e. The escape velocity from the surface of a planet whose mass and radius are three times those of the earth, will be
a) v_e b) $3v_e$ c) $9v_e$ d) $1/3v_e$
(a)

Escape velocity on surface of earth $v_e = \sqrt{\frac{2GM_e}{R_e}} \propto \sqrt{\frac{M_e}{R_e}}$

$$\therefore \frac{v_e}{v_p} = \sqrt{\frac{M_e}{R_e}} \times \sqrt{\frac{R_p}{M_p}} = \sqrt{\frac{M_e}{R_e}} \times \sqrt{\frac{3R_e}{3M_e}} = \frac{1}{1} = 1$$

or, $v_P = v_e$

11. A body weighs 72 N on the surface of the earth. What is the gravitational force on it due to earth at a height equal to half the radius of the earth from the surface?
a) 32 N b) 28 N c) 16 N d) 72 N
(a)
Mg = 72 N (body weight on the surface)

$$g = \frac{GM}{R^2}$$

At a height h = R/2, $g' = \frac{GM}{\left(R + \frac{R}{2}\right)^2} = \frac{4GM}{9R^2}$

Body weight at height H = $\frac{R}{2}$, mg' = m × $\frac{4}{9}\frac{GM}{R^2}$ $= m \times \frac{4}{9} x\, g = \frac{4}{9} mg$ $= \frac{4}{9} x\, 72 = 32\, N$

12. The potential energy of a satellite, having mass m and rotating at a height of 6.4×10^6 m from the earth surface, is

a) $-mgR_e$ b) $-0.67\ mgR_e$ c) $-0.5\ mgR_e$ d) $-0.33\ mgR_e$

(c)

Mass of the satellite = m and height of satellite from earth (h) = 6.4×10^6 m.

W.k.t gravitational potential energy of the satellite at height h

$U = -\frac{GM_e m}{R_e + h} = -\frac{gR_e^2 m}{2R_e} = -\frac{gR_e m}{2} = -0.5\, mgR_e$ (where, $GM_e = gR_e^2$ and $h = R_e$)

13. With what velocity should a particle be projected so that its height becomes equal to radius of earth?

a) $\left(\frac{GM}{R}\right)^{1/2}$ b) $\left(\frac{8GM}{R}\right)^{1/2}$ c) $\left(\frac{2GM}{R}\right)^{1/2}$ d) $\left(\frac{4GM}{R}\right)^{1/2}$

(a)

From conservation of energy, $\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}m\, x(0)^2 - \frac{GMm}{R+R}$

$\Rightarrow u^2 = \frac{2GM}{R} - \frac{2GM}{2R} = \frac{GM}{R}$

$\Rightarrow u = \sqrt{\frac{GM}{R}}$

14. Assuming the radius of the earth as R, the change in gravitational potential energy of a body of mass m, when it is taken from the earth's surface to a height 3R above its surface, is

a) 3 mg R b) $\frac{3}{4}$ mg R c) mg R d) $\frac{3}{2}$mg R

(b)

Gravitational potential energy (GPE) on the surface of earth, $U_1 = -\frac{GMm}{R}$

GPE at 3R, $U_2 = -\frac{GMm}{(R+3R)} = -\frac{GMm}{4R}$

∴ Change in GPE $= U_2 - U_1 = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R} = \frac{3gR^2m}{4R} = \frac{3}{4} mg\ R$ $\left(\because g = \frac{GM}{R^2}\right)$

15. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to height of 2m on the surface of A. What is the height of jump by the same person on the planet B?

a) $\frac{2}{3}$m b) $\frac{2}{9}$m c) 18 m d) 6 m

(c)

Applying conservation of total mechanical energy principle

$\frac{1}{2}mv^2 = mg_A h_A = mg_B h_B \Rightarrow g_A h_A = g_B h_B$

$\Rightarrow h_B = \left(\frac{g_A}{g_B}\right) h_A = 9 \times 2 = 18m$

16. Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the acceleration due to gravity on the surface of earth is g and that on the surface of the new planet is g', then

a) g' = g/9 b) g' = 27g c) g' = 9g d) g' = 3g

(d)

$g = \frac{4}{3}\pi G R \rho$

$\frac{g'}{g} = \frac{R'}{R} = \frac{3R}{R} = 3$

∴ g' = 3g

17. For a satellite moving in an orbit around the earth, the ratio of kinetic energy to potential energy is

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) 2 d) $\sqrt{2}$

(a)

$K = \frac{GMm}{2r}$

P.E. of satellite, $U = \frac{GMm}{r} \Rightarrow \frac{K}{U} = \frac{\frac{GMm}{2r}}{\frac{GMm}{r}} = \frac{1}{2}$

18. The Earth is assumed to be sphere of radius R. A platform is arranged at a height R from the surface of the Earth. The escape velocity of a body from this platform is f v, where v is its escape velocity from the surface of the Earth. The value of f is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\sqrt{2}$

(a)

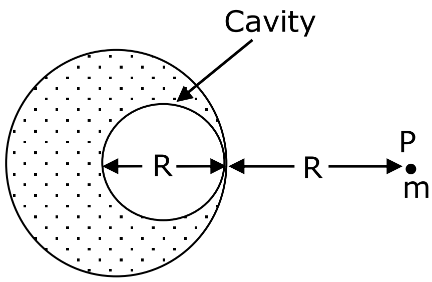
Escape velocity from the surface of earth is $v = \sqrt{\frac{2GM}{R}}$

Escape velocity at an altitude h from the earth is $v_e = \sqrt{\frac{2GM}{R+h}}$

When h = R, $v_e = \sqrt{\frac{2GM}{R+R}} = \sqrt{\frac{GM}{R}} = \frac{v}{\sqrt{2}} = fv$

Comparing it with given equation, $f = \frac{1}{\sqrt{2}}$

19. A uniform sphere of radius R and mass M exerts a force F on a body of mass m placed at point P at a distance 2R from the centre of the sphere. If a spherical cavity of radius R/2 is made in the sphere as shown in figure the force of attraction exerted by the sphere with cavity in it on the same body placed at the same point P will now be



a) $\frac{3F}{5}$ b) $\frac{5F}{7}$

c) $\frac{7F}{9}$ d) $\frac{F}{2}$

(c)

$F = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

Mass per unit volume = $\frac{M}{\frac{4\pi}{3}R^3}$

$\therefore$ Mass removed to make the cavity is M' = $\frac{M}{\frac{4\pi}{3}R^3} \times \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 = \frac{M}{8}$

The force of attraction on the body at P now will be F' = $f_{sphere} - f_{removed\ part}$

$$= \frac{GMm}{4R^2} - \frac{Gm \times \frac{M}{8}}{\left(\frac{3R}{2}\right)^2} = \frac{GMm}{4R^2} - \frac{GMm}{18R^2} = \frac{7}{36}\frac{GmM}{R^2} = \frac{7}{9} \times \frac{GmM}{4R^2} = \frac{7F}{9}$$

20. A block of mass m is hung on a spring of spring constant k and of negligible mass. The spring extends by x on the surface of the earth. When taken to a height h = R/2, the spring will extend by (R = radius of earth)

a) x b) $\frac{2x}{3}$ c) $\frac{3x}{8}$ d) $\frac{4x}{9}$

(d)

$\frac{GmM}{R^2} = mg$ (M = mass of earth)

But mg = kx

Therefore $\frac{GmM}{R^2} = kx \Rightarrow x = \frac{GMm}{kR^2}$

At a height h, $\frac{GmM}{(R+h)^2} = mg' = kx' \Rightarrow x' = \frac{GmM}{k(R+h)^2}$

From (i) and (ii) $x' = x\left(\frac{R}{R+h}\right)^2 = x\left(\frac{R}{R+R/2}\right)^2 = \frac{4x}{9}$

21. Infinite number of tiny spheres, each of mass m are placed on the x- axis at distance r, 2r, 4r,… etc from origin O. The magnitude of gravitational field intensity at O is

a) $\frac{4Gm}{3r^2}$ b) $\frac{2Gm}{3r^2}$ c) $\frac{Gm}{r^2}$ d) infinity

(a)

Since gravitational force is attractive, the direction of individual intensities will be towards O. Therefore, their magnitudes add up. Thus

$E = E_1 + E_2 + E_3 + \ldots = \frac{Gm}{r^2} + \frac{Gm}{(2r)^2} + \frac{Gm}{(4r)^2} + \ldots = \frac{Gm}{r^2}\left(1 + \frac{1}{2^2} + \frac{1}{4^2} + \ldots\right)$

$= \frac{Gm}{r^2}\left(\frac{1}{2^0} + \frac{1}{2^2} + \frac{1}{2^4} + \ldots\right) = \frac{Gm}{r^2} \times \frac{1}{\left(1 - \frac{1}{2^2}\right)} = \frac{4Gm}{3r^2}$

22. Two particles, each of mass m, are moving in a circle of radius r under the action of their mutual gravitational force. The speed of each particle round the circle will be proportional to

a) $\frac{1}{\sqrt{r}}$ b) $\frac{1}{r}$ c) $\frac{1}{r^{3/2}}$ d) $\frac{1}{r^2}$

(a)

The centripetal force for circular motion is provided by the gravitational force between the particles, since the gravitational force is attractive, the particles will be diametrically opposite to each other. If the speed of each particle is v, then for a given particle $\frac{mv^2}{r} = \frac{Gm^2}{(2r)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{r}}$

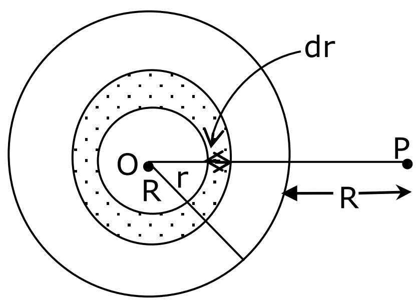
So $v \propto \frac{1}{\sqrt{r}}$

23. The density of a solid sphere of mass M and radius R varies with distance r from its centre as $\rho = \frac{k}{r}$ where k is a constant. The gravitational field due to this sphere at a distance 2R from its centre is given by

a) $\frac{\pi k G}{2}$ b) $\frac{\pi^2 k G}{R}$ c) $\frac{kG}{MR^2}$ d) $\frac{3}{2}\frac{MG}{k}$

(a)

Refer to Fig



To find gravitational field at point P, we divide the sphere into concentric shells of a very small thickness dr. The mass of each tiny shell can be assumed to be concentrated at centre O. Thus the mass M of the whole sphere can be assumed to be at O. The gravitational field due to this mass

M at point P is E = $\frac{GM}{(2R)^2} = \frac{GM}{4R^2}$ ---(1)

To find M, consider a shell of radius r and thickness dr. The volume of this shell is
dV = $(4\pi r^2)$ dr

Therefore, the mass of this shell is dM = ρ dV = $\left(\frac{k}{r}\right) x (4\pi r^2 \, dr)$ = 4 π kr dr

The mass of the whole sphere is M = $\int_0^R dM = 4\pi k \int_0^R r \, dr = 4\pi k \frac{R^2}{2} = 2\pi k R^2$ ---(2)

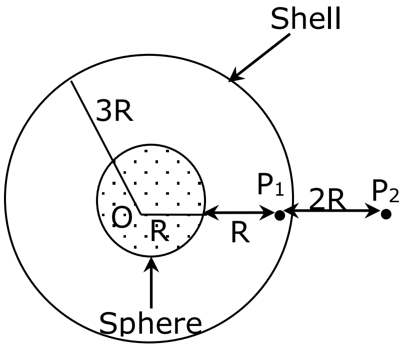
Using (ii) in (i), we get E = $\frac{G}{4R^2} x 2\pi k R^2 = \frac{\pi k G}{2}$

24. A solid sphere of mass M and radius R is surrounded by a concentric shell of equal mass m and radius 3R. The gravitational field at a point P_1 at a distance r_1 = 2R from the centre is I_1 and a point P_2 at a distance r_2 = 4R from the centre is I_2. The ratio $\frac{I_2}{I_1}$ is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{9}$ d) $\frac{9}{25}$

(a)

Refer to fig



Since point P_1 is inside the shell, the gravitational field at P_1 due to the shell is zero. The field at P_1 due to the solid sphere is $I_1 = \frac{GM}{r_1^2} = \frac{GM}{(2R)^2} = \frac{GM}{4R^2}$

Point P_2 is outside the shell as well as the sphere.
The mass M of the sphere and the mass M of the shell can be assumed to be concentrated at O so that the total mass at O is 2M.

The gravitational field due to mass 2M at point P_2 is $I_2 = \frac{G(2M)}{r_2^2} = \frac{2GM}{(4R)^2} = \frac{GM}{8R^2}$

$\Rightarrow \frac{I_2}{I_1} = \frac{1}{2}$

25. If the radius of the earth suddenly decreases to 80% of its present value, the mass of the earth remaining the same, the value of the acceleration due to gravity will

a) remain unchanged b) become (9.8 × 0.8) ms^{-2}
c) increase by 36% d) increase by about 56%

(d)

Now g = $\frac{GM}{R'^2}$

If R reduces to R' = 0.8 R, the value of g becomes g' = $\frac{GM}{R'^2} = \frac{GM}{0.64R^2} = \frac{g}{0.64} = \frac{9.8}{0.64}$

Increase in value of g = $\frac{g}{0.64} - g = \frac{0.36g}{0.64}$

∴ Percentage increase = $\frac{0.36g}{0.64g} x 100 = 56.25\%$

26. A small planet is revolving around very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force between the planet and the star were proportional to $R^{-5/2}$, then T would be proportional to

a) $R^{3/2}$ b) $R^{3/5}$ c) $R^{7/2}$ d) $R^{7/4}$

(d)

Since the gravitational force provides the necessary centripetal force for circular motion, we have $\frac{mv^2}{R} \propto R^{-5/2}$

Or $\frac{mv^2}{R} = kR^{-5/2}$, where k is constant

Therefore $v = \sqrt{\frac{kR^{-3/2}}{m}} \sqrt{\frac{k}{m}} R^{-3/4}$

Period of revolution $T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{m}{k}} \times R^{7/4}$ or $T \propto R^{7/4}$

27. The angular momentum of the earth revolving round the sun is proportional to R^n where R is the distance between the earth and the sun. The value of n is

a) 0.5 b) 1.0 c) 1.5 d) 2.0

(a)

Orbital velocity $v = \sqrt{\frac{GM}{R}}$

Now, angular momentum $L = mvR = m\sqrt{\frac{GM}{R}} R = m\sqrt{GM}\, R^{1/2}$ Or $L \propto R^{1/2}$

28. A satellite is moving around the earth in a stable circular orbit. Which one of the following statements will be wrong for such a satellite?

a) It is moving at a constant speed

b) Its angular momentum remains constant

c) It is acted upon by a force directed away from the centre of the earth which counter- balances the gravitational pull of the earth

d) It behaves as if it were a freely falling body.

(c)

Choices (a), (b) and (d) are all correct. Hence the choice (c)

29. An extremely small and dense neutron star of mass M and radius R is rotating at an angular frequencyω. If an object is placed at its equator, it will remain stuck to it due to gravity if

a) $M > \frac{R\omega}{G}$ b) $M > \frac{R^2\omega^2}{G}$ c) $M > \frac{R^3\omega^2}{G}$ d) $M > \frac{R^2\omega^3}{G}$

(c)

An object of mass m, placed at the equator of the star will experience two forces.

(i) an attractive force due to gravity towards the centre of the star

(ii) an outward centrifugal force due to the rotation of the star.

Force on object due to gravity is $F_g = \frac{GmM}{R^2}$

Centrifugal force on the object is $F_c = mR\omega^2$

The object will remain stuck to the star and not fly off if $F_g > F_c$

or $\frac{GmM}{R^2} > mR\omega^2$ or $M > \frac{R^3\omega^2}{G}$

30. The escape velocity of a body on the earth's surface is v_e. A body is thrown with a speed $3v_e$. Assuming that the sun and planets do not influence the motion of the body, its speed at infinity would be

a) zero b) v_e c) $\sqrt{2}\,v_e$ d) $2\sqrt{2}\,v_e$

(d)

If v_i and v_f are respectively the initial and final speeds of the body, we have, from the law of conservation of energy $\frac{1}{2}mv_i^2 - \frac{GmM}{R} = \frac{1}{2}mv_f^2$ (i)

where m is the mass of the body and M is the mass of the earth R its radius. The escape velocity is given by $\frac{1}{2}mv_e^2 = \frac{GmM}{R}$ --- (ii)

Using (ii) in (i) gives $\frac{1}{2}mv_i^2 - \frac{1}{2}mv_e^2 = \frac{1}{2}mv_f^2$ or $v_f = \left(v_i^2 - v_e^2\right)^{1/2}$

Given $v_i = 3v_e$.

Therefore, $v_f = 2\sqrt{2}\, v_e$

31. What is the minimum energy required to launch a satellite of mass m from the surface of the earth of radius R in a circular orbit at an altitude of 2R?

a) $\frac{5GmM}{6R}$ b) $\frac{2GmM}{3R}$ c) $\frac{GmM}{2R}$ d) $\frac{GmM}{3R}$

(a)

Gravitational potential energy U = $-\frac{GmM}{r}$ where M is the mass of the earth

K = $\frac{GmM}{2r}$

The total energy of the satellite in orbit is E = U + K = $-\frac{GmM}{2r}$

It is given that r = 2R +R = 3R, where R is the radius of the earth.

$\therefore$ E = $-\frac{GmM}{6R}$

Now PE on the surface of the earth U = $-\frac{GmM}{R}$

$\therefore$ Minimum energy required (E_{min})= E +U = $-\frac{GmM}{6R} - \left(-\frac{GmM}{R}\right) = \frac{5GmM}{6R}$

32. Two stars, each of mass m and radius R are approaching each other for a head-on collision. They start approaching each other when their separation is r >> R. If their speeds at this separation are negligible, the speed with which they collide would be

a) $v = \sqrt{Gm\left(\frac{1}{R} - \frac{1}{r}\right)}$ b) $v = \sqrt{Gm\left(\frac{1}{2R} - \frac{1}{r}\right)}$

c) $v = \sqrt{Gm\left(\frac{1}{R} + \frac{1}{r}\right)}$ d) $v = \sqrt{Gm\left(\frac{1}{2R} + \frac{1}{r}\right)}$

(b)

The speeds of stars at separations r are negligible, therefore their energy is entirely potential at this separation (since KE = 0)

E_1 = (PE at r) = $-\frac{Gm_1m_2}{r} = -\frac{Gm^2}{r}$

As the stars approach each other under gravitational attraction, they begin to acquire speed and hence kinetic energy at the expense of potential energy. When they eventually collide, the separation between their centres is r = R + R = 2R

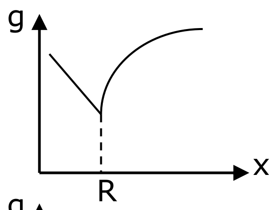
At = 2R, the total energy is E_2 = PE at (r = 2R) + KE at (r = 2R) = $-\frac{Gm^2}{2R} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

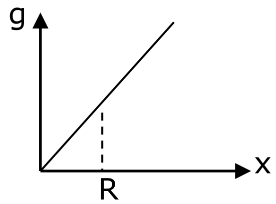
Or $E_2 = -\frac{Gm^2}{2R} + mv^2$

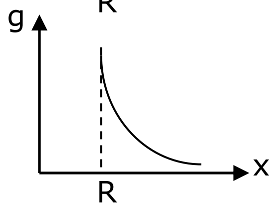
From the principle of conservation of energy, $E_1 = E_2$, i. e. $-\frac{Gm^2}{r} = -\frac{Gm^2}{2R} + mv^2$

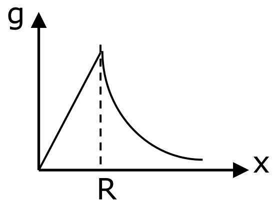
v = $\sqrt{Gm\left(\frac{1}{2R} - \frac{1}{r}\right)}$

33. Variation of acceleration due to gravity (g) with distance x from the centre of the earth is best represented by (R $\rightarrow$ Radius of the earth)

a) 

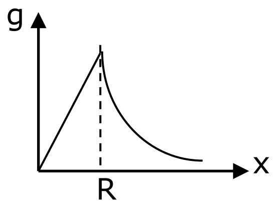
b) 

c) 

d) 

(d)

Acceleration due to gravity $g = \begin{cases} \dfrac{GM}{R^3}x & ; \quad x < R \\ \dfrac{GM}{x^2} & ; \quad x \geq R \end{cases}$



34. At what distance from the centre of the moon is the point at which the strength of the resultant field of earth's and moon's gravitational fields equal to zero? The earth's mass is 81 times that of moon and the distance between centres of earth and moon is 60 R where R is the radius of earth.

a) 6 R b) 4 R c) 3 R d) 5 R

(a)

Let the gravitational field strength be zero at point P having distance x from the moon. Then,

$I_{earth} = I_{moon}$

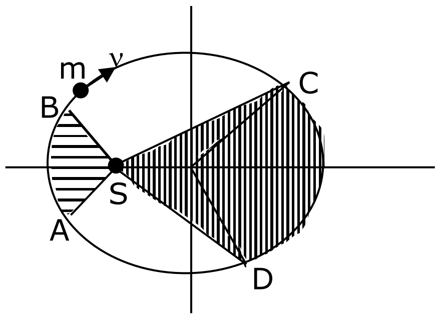
$$\frac{GM_E}{(60R - x)^2} = \frac{GM_M}{x^2}$$

$$\frac{81}{(60R - x)^2} = \frac{1}{x^2}$$

$$\frac{9}{60R - x} = \frac{1}{x}$$

$x = 6R$

35. The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then



a) $t_1 = 4t_2$ b) $t_1 = 2t_2$ c) $t_1 = t_2$ d) $t_1 > t_2$

(b)

Equal areas are swept in equal time. t_1, the time taken to go from C to D = $2t_2$ where t_2 is the time taken to go from A to B. As it is given that area SCD = 2 area SAB.

36. At what height from the surface of earth the gravitational potential and the value of g are -5.4×10^7 J kg^{-2} and 6.0 ms^{-2} respectively? Take the radius of earth as 6400km.

a) 1400km b) 2000km c) 2600km d) 1600km

(c)

Gravitation potential at a height h from the surface of earth, $V_h = -5.4 \times 10^7$ Jkg^{-2}

At the same point acceleration due to gravity, $g_h = 6ms^{-2}$

$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

We know, $V_h = -\frac{GM}{(R+h)}, g_h = \frac{GM}{(R+h)^2} = -\frac{V_h}{R+h}$

$\Rightarrow \quad R + h = -\frac{V_h}{g_h}$

$\therefore \quad h = -\frac{V_h}{g_h} - R = -\frac{(-5.4 \times 10^7)}{6} - 6.4 \times 10^6 = 9 \times 10^6 - 6.4 \times 10^6 = 2600$ km

37. A satellite which is geostationary in a particular orbit is taken to another orbit. Its distance from the centre of earth in the new orbit is 2 times that of the earlier orbit. The time period in the second orbit is:

a) 48 hours b) $48\sqrt{2}$ hours c) 24 hours d) $24\sqrt{2}$ hours

(b)

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{2r_1}{r_1}\right)^3$$

$T_2 = 2\sqrt{2}\ T_1 = 2\sqrt{2} \times 24 = 48\sqrt{2}$ hrs

38. The ratio of escape velocity at earth v_e to the escape velocity at a planet v_p whose radius and mean density are twice as that of earth is

a) 1:4 b) 1: $\sqrt{2}$ c) 1:2 d) 1: $2\sqrt{2}$

(d)

As escape velocity, $v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \cdot \frac{4\pi R^3}{3}\rho} = R\sqrt{\frac{8\pi G}{3}\rho}$

$\therefore \quad \frac{v_e}{v_p} = \frac{R_e}{R_p} \times \sqrt{\frac{\rho_e}{\rho_p}} = \frac{1}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}}$ ($\because$ $R_p = 2R_e$ and $\rho_p = 2\rho_e$)

39. The ratio of the energy required to raise a satellite upto a height h above the earth of radius R to the kinetic energy of the satellite in that orbit is

a) R:h b) h :R c) R : 2h d) 2h:R

(d)

Energy required to raise the satellite to a height h from surface of earth is given by

$$U = -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right) = \frac{GMmh}{R(R+h)}$$

Kinetic energy of satellite is given by $K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\frac{GM}{(R+h)}$

$\therefore \quad \frac{U}{K} = \frac{2h}{R}$

40. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface , is

a) $\frac{mg_0R^2}{2(R+h)}$ b) $-\frac{mg_0R^2}{2(R+h)}$ c) $\frac{2mg_0R^2}{R+h}$ d) $-\frac{2mg_0R^2}{R+h}$

(b)

Total energy of satellite at height h from the earth surface.

$E = PE + KE = -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2$ …(i)

Also, $\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$ or $v^2 = \frac{GM}{R+h}$ … (ii)

From eqns. (i) and (ii), $E = -\frac{GMm}{(R+h)} + \frac{1}{2}\frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$

$= -\frac{1}{2}\frac{GM}{R^2} \times \frac{mR^2}{(R+h)} = -\frac{mg_0R^2}{2(R+h)}$ $\left(\because g_0 = \frac{GM}{R^2}\right)$

41. A satellite is orbiting the earth at a height of 5R above the surface of the earth has a time period of 24 hrs, R being the radius of the earth. The time period of another satellite in hours at a height of 2R from the surface of the earth is

a) 5 b) 10 c) $6\sqrt{2}$ d) $\frac{6}{\sqrt{2}}$

(c)

According to Kepler's third law $T \propto r^{3/2}$

$\therefore \frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{R+2R}{R+5R}\right)^{3/2} = \frac{1}{2^{3/2}}$

Or $\frac{T_2}{24} = \frac{1}{2^{3/2}}$ or $T_2 = \frac{24}{2^{3/2}} = \frac{24}{2\sqrt{2}} = 6\sqrt{2}$ hours

42. The imaginary angular velocity of the earth for which the effective acceleration due to gravity at the equator shall be zero is equal to

a) 1.25×10^{-3} rad/s b) 2.50×10^{-3} rad/s c) 3.75×10^{-3} rad/s d) 5.0×10^{-3} rad/s

(a)

The effective acceleration due to gravity at the equator is $g' = g - R\omega^2$

As per question, $g' = 0$

$\therefore \omega = \sqrt{\frac{g}{R}}$

Substituting the given values, we get $\omega = \sqrt{\frac{10}{6400 \times 10^3}} = \frac{1}{8} \times 10^{-2}$ rad/s $= 1.25 \times 10^{-3}$ rad/s.

43. A body is projected up from the surface of the earth with velocity $\left(\frac{3}{4}\right)^{th}$ of its escape velocity. If R be the radius of earth, the height it reaches is

a) $\frac{3R}{10}$ b) $\frac{9R}{7}$ c) $\frac{8R}{5}$ d) $\frac{9R}{5}$

(b)

Let the body be projected up from the surface of earth with velocity v and it reaches a height h.

By conservation of energy (relative to surface of earth), we get

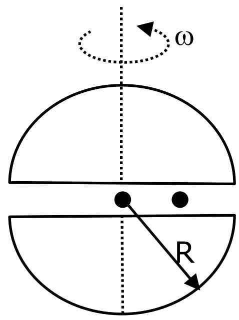
$$\frac{1}{2}mv^2 = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

According to problem, $v = \frac{3}{4}v_e = \frac{3}{4}\sqrt{2gR}$ $(\because v_e = \sqrt{2gR})$

$$\therefore \frac{1}{2} \times \frac{9}{16} \times 2gR = \frac{gh}{\left(1+\frac{h}{R}\right)}$$

or $\frac{9}{16} = \frac{h}{(R+h)}$ or $9R + 9h = 16h$ or $h = \frac{9R}{7}$

44. A straight smooth tunnel is dug through a spherical planet whose mass density ρ_0 is constant. The tunnel passes through the centre of the planet and is perpendicular to the planet's axis of rotation, which is fixed in space. The planet rotates with the angular velocity ω so that objects in the tunnel have no acceleration relative to the tunnel. The value of ω is



(a) $\omega = \sqrt{\frac{4}{3}\pi G\rho_0}$ (b) $\omega = \sqrt{\frac{2}{3}\pi G\rho_0}$

(c) $\omega = \sqrt{\pi G\rho_0}$ (d) $\omega = \sqrt{\frac{\pi G\rho_0}{3}}$

(a)

Gravitational field at a distance 'r', $g' = r\omega^2$

$\left(\frac{Gm}{R^3}\right)r = r\omega^2$

$\frac{G\left(\frac{4}{3}\pi R^3\rho_0\right)}{R^3} = \omega^2$

$\omega = \sqrt{\frac{4}{3}\pi G\rho_0}$

45. The radii of circular orbits of two satellites A and B of the earth are 4R and R, respectively. If the speed of satellite A is 3v, then the speed of satellite B will be

(a) $\frac{3v}{4}$ (b) $6v$ (c) $12v$ (d) $\frac{3v}{2}$

(b)

The orbital speed of satellite $v = \sqrt{\frac{GM}{r}} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}} \Rightarrow \frac{3v}{v_B} = \sqrt{\frac{R}{4R}} \Rightarrow v_B = 6v$.

46. If a man weighs 90N on the surface of earth, the height above the surface of the earth of radius R, where the weight is 30N is

a) 0.73R b) $\frac{R}{\sqrt{3}}$ c) $\frac{R}{3}$ d) $\sqrt{3}$ R

(a)

Acceleration due to gravity at a height h above the surface of the earth is g' = $\frac{gR^2}{(R+h)^2}$ …(i)

where g is the acceleration due to gravity on the surface of the earth and R is the radius of the earth.
Weight of man on the surface of the earth is W = mg …(ii), where m is the mass of the man.
Weight of the same man at a height h above the surface of the earth is

W' = mg' = $\frac{mgR^2}{(R+h)^2}$ (Using (i)) …(iii)

Divide (iii) by (ii), we get $\frac{W'}{W} = \frac{R^2}{(R+h)^2}$

As per question, $\frac{W'}{W} = \frac{30N}{90N} = \frac{1}{3}$

$\therefore \frac{1}{3} = \frac{R^2}{(R+h)^2}$ or $(R + h)^2 = 3R^2$ or $R + h = \sqrt{3}R$ or $h = (\sqrt{3} - 1)R = 0.73R$

47. A body of mass M is divided into two parts m and M – m. The gravitational force between them is maximum, if $\frac{m}{M}$ is

a) 1:1 b) 1:2 c) 1:3 d) 1:4

(b)

Let r be the distance between m and (M – m). The gravitational force between them is

$$F = \frac{Gm(M-m)}{r^2} = \frac{G(mM-m^2)}{r^2}$$

For F to be maximum, $\frac{dF}{dm} = 0$

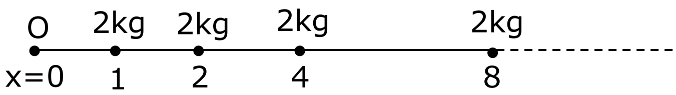
$$\therefore \quad \frac{d}{dm}\left(\frac{G(mM-m^2)}{r^2}\right) = 0 \text{ or } M - 2m = 0 \text{ or } \frac{m}{M} = \frac{1}{2}$$

48. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1m, 2m, 4m, 8m, ….., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be

a) $-\frac{4}{3}G$ b) -4G c) – G d) $-\frac{8}{3}G$

(b)

The resulting gravitational potential at the origin O due to each of mass 2kg located at positions as shown in figure is



$$V = \frac{Gm}{r}$$

$$V = -\frac{G \times 2}{1} - \frac{G \times 2}{2} - \frac{G \times 2}{4} - \frac{G \times 2}{8} - \ldots$$

$$= -2G\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right] = -2G\left[\frac{1}{1-\frac{1}{2}}\right] = -2G\left[\frac{2}{1}\right] = -4G$$

49. A satellite moving round the earth in a circular orbit of radius r and speed ν suddenly loses some of its energy. Then

a) r will increase and ν will decrease
b) both r and ν will decrease
c) both r and ν will increase
d) r will decrease and ν will increase

(d)

Total energy of the satellite = $\frac{1}{2}m\nu^2 - \frac{GMm}{r} = -\frac{GMm}{2r}$

Also, $\frac{m\nu^2}{r} = \frac{GMm}{r^2}$ or $\nu = \frac{1}{\sqrt{r}}$

If the energy of satellite decreases by some amount, then the value of r decreases, ν will increase.

50. If the change in the value of 'g' at a height h above the surface of the earth is the same as at a depth x below it, then (both x and h being much smaller than the radius of the earth):

a) x = h b) x = 2h c) x = 1/2h d) $x = h^2$

(b)

$g_h = g_x$

$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{x}{R}\right) \quad (\because x \ll R)$$

$\therefore \quad x = 2h$