1 PUC: CHAPTER- 03

MOTION IN A STRAIGHT LINE

1. A body is projected vertically upwards with a velocity u. It crosses a point in its journey at a height h twice, just after 1s and 7s. The value of u in ms^{-1} is ($g = 10ms^{-2}$)

(a) 50 (b) 40 (c) 30 (d) 20

(c)

$h = ut_1 - \frac{1}{2} gt_1^2$

$h = ut_2 - \frac{1}{2} gt_2^2$

$u(t_2 - t_1) = \frac{1}{2} g(t_2^2 - t_1^2)$

Using $u = \frac{1}{2} g\left(t_2 + t_1\right) = \frac{1}{2} 10(7+1) = 40$

2. A body dropped from a height h with initial velocity zero, strikes the ground with a velocity of 3 m/s. Another body of same mass is dropped from the height h with an initial velocity of 4 m/s. Find the velocity of second mass with which it strikes the ground.

(a) 3 m/s (b) 4 m/s (c) 5 m/s (d) 12 m/s

(c)

For first body: $v^2 = u^2 + 2gh$

$(3)^2 = 0 + 2gh$

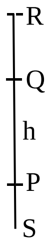
For second body: $v^2 = (4)^2 + 2gh = 4^2 + 3^2 = 25$

$v = 5$ m/s

3. A stone projected upwards with a velocity u reaches two points P and Q separated by a distance h with velocities u/2 and u/3 respectively. The maximum height reached by it is

(a) $\frac{9h}{5}$ (b) $\frac{18h}{5}$ (c) $\frac{36h}{5}$ (d) $\frac{72h}{5}$

(c)



Maximum height $H = \frac{u^2}{2g}$

For PQ, $\frac{u^2}{9} - \frac{u^2}{4} = -2gh \Rightarrow -\frac{5u^2}{36} = -2gh$

$H = \frac{u^2}{2g} = \frac{36h}{5}$

4. A ball is thrown vertically upwards from the ground. If T_1 and T_2 are the respective time taken in going up and coming down, and the air resistance is not ignored, then

(a) $T_1 > T_2$ (b) $T_1 = T_2$ (c) $T_1 < T_2$ (d) noting can be said

(c)

Air resistance (let F) is always opposite to motion (or velocity)

Retardation in upward journey $a_1 = \frac{F + mg}{m} = g + \frac{F}{m}$

Acceleration in downward journey $a_2 = \frac{mg - F}{m} = g - \frac{F}{m}$

Since, $a_1 > a_2 \Rightarrow T_1 < T_2$

5. A particle moves along a straight line. Its position at any instant is given by $x = 32t - \frac{8t^3}{3}$ where x is in metres and t in second. Find the acceleration of the particle at the instant when particle is at rest.
(a) -16 ms^{-2} (b) -32 ms^{-2} (c) 32 ms^{-2} (d) 16 ms^{-2}

(b)
$v = \frac{dx}{dt} = 32 - 8t^2$
$v = 0$ at $t = 2$ s
$a = \frac{dv}{dt} = -16t$
At $t = 2s, a = -32 m/s^2$

6. A stone is dropped from the top of a tower and one second later, a second stone is thrown vertically downward with a velocity 20 ms^{-1}. The second stone will overtake the first after travelling a distance of (g=10 ms^{-2})
(a) 13 m (b) 15 m (c) 11.25 m (d) 19.5 m

(c)
When the second stone overtakes first, displacement for both is same.
$0 + \frac{1}{2}gt^2 = 20(t-1) + \frac{1}{2}g(t-1)^2$ using $s = ut + \frac{1}{2}at^2$
$\frac{1}{2}gt^2 = 20t - 20 + \frac{1}{2}gt^2 - gt + \frac{1}{2}g$
10t = 15
$\therefore$ t = 1.5 s
h = ½ gt^2 = 0.5 × 10 × 2.25 = 0.5 × 22.5 = 11.25 m

7. Two identical stones are shot upward one after another at an interval of 2s along the same vertical line with same initial velocity of 40 ms^{-1}. The height at which the stones collide is (g = 10 ms^{-2})
(a) 50 m (b) 75 m (c) 100 m (d) 125 m

(b)
When the two stones collide, $h_1 = h_2$
$\therefore (40)t - \frac{1}{2} \times 10 \times t^2 = (40)(t-2) - \frac{1}{2} \times 10 \times (t-2)^2$ using $s = ut + \frac{1}{2}at^2$
$40t - 5t^2 = 40t - 80 - 5t^2 + 20t - 20$
20t = 100
t = 5 s,
Then $h_1 = (40)(5) - \frac{1}{2} \times 10 \times (5)^2 = 75$ m

8. A moving car possesses average velocities of 5 ms^{-1}, 10 ms^{-1} and 15 ms^{-1} in the first, second and third seconds, respectively. What is the total distance covered by the car in these 3s?
(a) 15 m (b) 30 m (c) 55 m (d) 10 m

(b)
Distance covered $s = v_{av} \times time$
In first second ; $s_1 = 5 \times 1 = 5$ m
In second second: $s_2 = 10 \times 1 = 10$ m

In third second $s_3 = 15 \times 1 = 15$ m

Total distance travelled, $s = s_1 + s_2 + s_3 = 5 + 10 + 15 = 30$ m

9. Two trains, each travelling with a speed of 37.5 kmh^{-1}, are approaching each other on the same straight track. A bird that can fly at a constant speed of 60 kmh^{-1} flies off from one train when they are 90 km apart and heads directly for the other train. On reaching the other train, it flies back to the first and so on. Total distance covered by the bird is

(a) 90 km (b) 54 km (c) 36 km (d) 72 km

(d)

Relative speed of trains = 37.5 + 37.5 = 75 kmh^{-1}

Time taken by the trains to meet = 90/75 = 6/5 hr

Since speed of bird is 60 kmh^{-1}, distance travelled by the bird = $60 \times \frac{6}{5} = 72$ km

10. A train is moving at constant speed V when its driver observes another train in front of him on the same track and moving in the same direction with constant speed v. If the distance between the trains is x, then what should be the minimum retardation of the train so as to avoid collision?

(a) $\frac{(V+v)^2}{x}$ (b) $\frac{(V-v)^2}{x}$ (c) $\frac{(V+v)^2}{2x}$ (d) $\frac{(V-v)^2}{2x}$

(d)

Here relative velocity of the train w.r.t other train is $V - v$

Hence, $0 - (V-v)^2 = 2ax$ using $v^2 = u^2 + 2as$

$$a = -\frac{(V-v)^2}{2x}$$

11. Two objects are moving along the same straight line. They cross a point A with an acceleration α, 2 α and velocity 2u, u at time t = 0. The distance moved by the object when one overtakes the other is

(a) $\frac{6u^2}{\alpha}$ (b) $\frac{2u^2}{\alpha}$ (c) $\frac{4u^2}{\alpha}$ (d) $\frac{8u^2}{\alpha}$

(a)

At the time of overtaking, $s_1 = s_2$

$$\therefore 2ut + \frac{1}{2}\alpha t^2 = ut + \frac{1}{2}(2\alpha)t^2$$

$$\therefore \quad t = \frac{2u}{\alpha}$$

$$\therefore \quad s_1 (\text{or } s_2) = (2u)\left(\frac{2u}{\alpha}\right) + \frac{1}{2}(\alpha)\left(\frac{2u}{\alpha}\right)^2 = \frac{6u^2}{\alpha}$$

12. The distance travelled by a particle starting from rest and moving with acceleration $\frac{4}{3} ms^{-2}$, in the third second is

(a) $\frac{10}{3}$ m (b) $\frac{19}{3}$ m (c) 6 m (d) 4 m

(a)

$$S_n = u + \frac{a}{2}(2n-1) \Rightarrow S = 0 + \frac{4/3}{2}(2 x 3 - 1)$$

$$\Rightarrow S_3 = \frac{10}{3} m$$

13. A man is 45 m behind the bus when the bus starts accelerating from rest with acceleration 2.5 m/s^2. With what minimum velocity should the man start running to catch the bus

(a) 12 m/s (b) 14 m/s (c) 15 m/s (d) 16 m/s

(c)

When the man catches the bus, $s_{man} = s_{bus}$

$ut = 45 + ½ at^2$

$ut = 1.25t^2 + 45$

$$\Rightarrow u = 1.25t + \frac{45}{t}$$

To find the minimum value of u, $\frac{du}{dt} = 0$

$$\frac{du}{dt} = 1.25 - \frac{45}{t^2} = 0$$

We get t = 6 sec then,

$$u = 1.25 \times 6 + \frac{45}{6} = 7.5 + 7.5 = 15 m/s$$

14. The displacement x of a particle varies with time t, $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

a) go on decreasing with t
b) be independent of α and β
c) drop to zero when $\alpha = \beta$
d) go on increasing with time

(d)

$x = ae^{-\alpha t} + be^{\beta t}$

Velocity v = $\frac{dx}{dt} = \frac{d}{dt}(ae^{-\alpha t} + be^{\beta t}) = ae^{-\alpha t}(-\alpha) + be^{\beta t}(\beta) = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$

Acceleration is positive so velocity goes on increases with time

15. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is x = 0 at t = 0, then its displacement after time t = 1s is

(a) $v_0 + 2g + 3f$ (b) $v_0 + g/2 + f/3$ (c) $v_0 + g + f$ (d) $v_0 + g/2 + f$

(b)

$v = v_0 + gt + ft^2$

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)dt$$

$$[x]_0^x = \left[v_o t + \frac{gt^2}{2} + \frac{ft^3}{3} \right]_0^1$$

$$x = v_0 + g\left(\frac{1}{2}\right) + f\left(\frac{1}{3}\right).$$

16. The relation 3t = $\sqrt{3}x+6$ describes the displacement of a particle in one direction where x is in metres and t in second. The displacement, when velocity is zero, is

(a) 24 m (b) 12 m (c) 5 m (d) Zero

(d)

$3t=\sqrt{3}x+6 \Rightarrow 3x=(3t-6)^2$

$\Rightarrow x=3t^2-12t+12$

$v=\frac{dx}{dt}=6t-12$

When $v=0, t=2\,s$

$x = 3(2)^2 - 12 \times 2 + 12 = 0$

17. P is the point of contact of a wheel and the ground. The radius of the wheel is 1 m. The wheel rolls on the ground without slipping. The displacement of point P when the wheel completes half rotation is

(a) 2 m (b) $\sqrt{\pi^2+4}\,m$ (c) π m (d) $\sqrt{\pi^2+2}\,m$

(b)

Displacement in horizontal direction = πR = π m.

Displacement in vertical direction = 2R = 2 m.

∴ Resultant displacement = $\sqrt{\pi^2+4}\,m$

18. Two trains A and B 100 km apart are travelling towards each other on different tracks with starting speed of 50 kmph for both. The train A accelerates at 18 $kmph^2$ and the train B retard at the rate 18 $kmph^2$. The distance covered by the train A when they cross each other is

(a) 45 km (b) 59 km (c) 65 km (d) None

(b)

Using $s=ut+\frac{1}{2}at^2$, For train A, $x=50t+\frac{1}{2}18t^2 \Rightarrow x=50t+9t^2 \rightarrow (1)$

For train B, $100-x=50t-\frac{1}{2}18t^2 \Rightarrow 100-x=50t-9t^2 \rightarrow (2)$

Eqn(1) + eqn (2) 100 = 100t

t = 1h.

Therefore, x = $x=50t+9t^2=59\,km$

19. A particle's position as a function of time is described as $y(t) = 2t^2 + 3t + 4$ (in m). What is the average velocity of the particle from t = 0 to t = 3 s?

(a) 3 m/s (b) 6 m/s (c) 9 m/s (d) 12 m/s

(c)

$y(t) = 2t^2 + 3t + 4$

y(0) = 4m, y(3) = 31m, s = 27 m

Average velocity = $\frac{\text{total distance}}{\text{total time}}=\frac{y_2-y_1}{t_2-t_1}=\frac{31-4}{3-0}=9\,m/s$

20. A train of 150 m length is going towards north direction at a speed of 10 ms^{-1}. A parrot flies at the speed of 5 ms^{-1} towards south direction parallel to the railway track. The time taken by the parrot to cross the train is

(a) 12 s (b) 10 s (c) 15 s (d) 8 s

(b)

Velocity of parrot w.r.t train $v_{PT} = 15 ms^{-1}$, therefore $t = \frac{s}{v_{PT}} = \frac{150}{15} = 10s$

21. A juggler keeps on moving four balls in air throwing the balls vertically upwards after regular intervals. When one ball leaves his hand (speed = 20 ms^{-1}) the position of other balls (height in metre) will be (take g= 10 ms^{-2})
(a) 10, 20, 10 (b) 15, 20, 15 (c) 5, 15, 20 (d) 5, 10, 20

(b)

Time taken by the same ball to return to the hands the juggler is T= $\frac{2u}{g} = \frac{2 \times 20}{10} = 4\,s$.

So he is throwing the balls after l s each.
Let at some instant he throws ball number 4. Before 1 s of throwing it, he throws ball 3.

So the height of ball 3 is $h_3 = 20 \times 1 - \frac{1}{2}10(1)^2 = 15\text{ m}$

Before 2 s, he throws ball 2. So the height of ball 2 is $h_2 = 20 \times 2 - \frac{1}{2}10(2)^2 = 20\text{ m}$

Before 3 s, he throws ball 1. So the height of ball 1 is $h_1 = 20 \times 3 - \frac{1}{2}10(3)^2 = 15\text{ m}$

22. A ball is dropped from a bridge 122.5 m high. After the first ball has fallen for 2 s, a second ball is thrown straight down after it. What must the initial velocity of the second ball be, so that both the balls hit the surface of water at the same time? (g = 9.8 ms^{-2})
(a) 49 m/s (b) 55.5 m/s (c) 26.1 m/s (d) 9.8 m/s

(c)

For the 1st ball: $h = \frac{1}{2}gt^2$ or $\frac{2 \times 122.5}{9.8} = t^2$ or t =5 s
Now the second ball has to travel a distance of 122.5 m in 3 s.

So $122.5 = u \times 3 + \frac{1}{2}9.8 \times 9$

Or 122.5 – 44.1 =3u
u = 26.1 m/s

23. A body of mass 3 kg falls from the multi storied building 100 m high and buries itself 2 m deep in the sand. The time of penetration will be: (g = 9.8 ms^{-2})
(a) 9 s (b) 0.9 s (c) 0.09 s (d) 10 s

(c)
$v^2 = u^2 + 2gh$ Or $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 100} = \sqrt{1960} = 14\sqrt{10}\text{m/s}$
This is the velocity with which the stone hits the ground.

When the stone penetrates, $\frac{s}{t} = \frac{u+v}{2}$

$\frac{2}{t} = \frac{14\sqrt{10}+0}{2}$

t = $\frac{2}{7\sqrt{10}} = \frac{2\sqrt{10}}{70}$ = 0.09 s

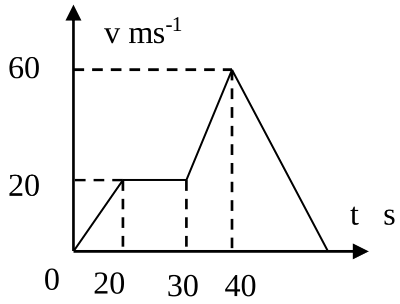
24. The velocity-time graph of a body is given in figure. The maximum acceleration in ms^{-2} is

(a) 4

(b) 3

(c) 2

(d) 1



(a)

Slope of v-t graph gives acceleration. $a = \frac{60-20}{40-30} = 4ms^{-2}$

25. A body travels 200 cm in the first 2 s and 220 cm in the next 4 s with deceleration. The velocity of the body at the end of the 7^{th} second is
(a) 5 cms^{-1} (b) 10 cms^{-1} (c) 15 cms^{-1} (d) 20 cms^{-1}

(b)
Use, $s = ut + ½ at^2$

$200 = 2u + \frac{1}{2}a2^2 \Rightarrow u + a = 100$ 1

$420 = 6u + \frac{1}{2}a6^2 \Rightarrow u + 3a = 70$ 2

Eqn 1 – Eqn 2 gives -2a = 30
a = - 15 cms^{-2} & u = 115 cms^{-1},
Using v = u + at = 115 - 15×7 = 10 cms^{-1}

26. A body A starts from rest with an acceleration a_1. After 2 s, another body B starts from rest with an acceleration a_2. If they travel equal distances in the 5^{th} second, after the start of A, then the ratio $a_1 : a_2$ is equal to:
(a) 5:9 (b) 5:7 (c) 9:5 (d) 9:7

(a)
Time taken by body A, t_1= 5 s
Acceleration of body A = a_1
Time taken by body B, t_2 = 5 – 2 = 3 s
Acceleration of body B = a_2

Distance covered by first body in 5^{th} second after start, $s_5 = u + \frac{a_1}{2}(2t_1 - 1) = 0 + \frac{a_1}{2}(2x5-1) = \frac{9a_1}{2}$

Distance covered by the second body in the 3^{rd} second after its start,

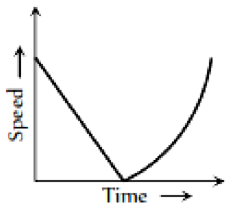
$s_3 = u + \frac{a_2}{2}(2t_2 - 1) = 0 + \frac{a_2}{2}(2 x 3 - 1) = \frac{5a_2}{2}$

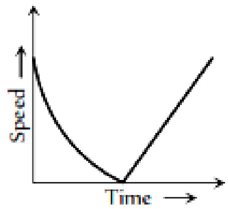
Since, $s_5 = s_3$

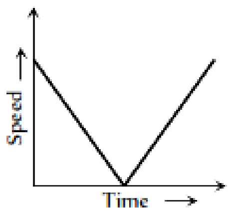
$\therefore \frac{9a_1}{2} = \frac{5a_2}{2}$

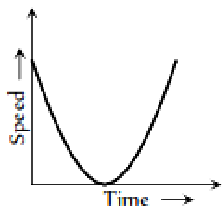
or $a_1 : a_2 = 5 : 9$

27. A ball is thrown vertically upwards. Which of the plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored?

(a) 

(b) 

(c) 

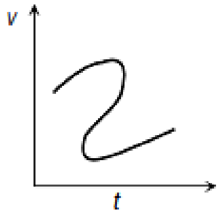
(d) 

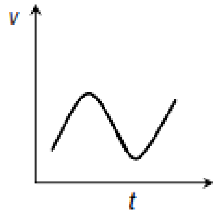
(c)

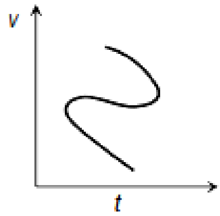
For upward motion effective acceleration = - (g+a), and for downward motion = (g - a)

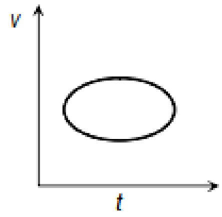
But both are constants. So the slope of speed-time graph will be constant.

28. Which of the following velocity-time graphs shows a realistic situation for a body in motion?

(a) 

(b) 

(c) 

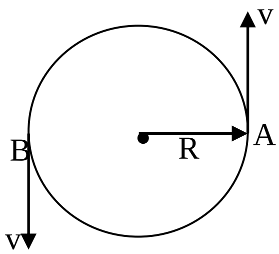
(d) 

(b)

Other graph shows more than one velocity of the particle at single instant of time which is not practically possible.

29. A particle is moving with a constant speed v in a circle. What is the magnitude of average velocity after half rotation?

(a) 2v (b) $2\frac{v}{\pi}$ (c) $\frac{v}{2}$ (d) $\frac{v}{2\pi}$



(b)

Let R be the radius of a circle.

Displacement = AB = 2R

Time taken to go from A to B, $t = \frac{\pi R}{v}$

Average velocity, $v_{av} = \frac{\text{Displacement}}{\text{Time taken}} = \frac{2R}{(\pi R / v)} = \frac{2}{\pi}v$

30. Two trains A and B, 100m and 60m long, are moving in opposite directions on parallel tracks. The velocity of the shorter train is 3 times that of the longer train. If the trains take 4s to cross each other, the velocities of the trains are

(a) V_A=10 ms^{-1}, V_B=30 ms^{-1} (b) V_A=2.5 ms^{-1}, V_B=7.5 ms^{-1}

(c) V_A=20 ms^{-1}, V_B=60 ms^{-1} (d) V_A=5 ms^{-1}, V_B=15 ms^{-1}

(a)

$v_B = 3v_A$

$s_{rel} = v_{rel} \times t \Rightarrow 160 = (v_A + v_B) \times 4$

$v_A + v_B = 40\ ms^{-1}$

31. Two trains one travelling at 15 ms^{-1} and other at 20 ms^{-1} are headed towards one another along a straight track. Both the drives apply brakes simultaneously when they are 500 m apart. If each train has a retardation of 1 ms^{-2}, the separation after they stop is:

(a) 192.5 m (b) 225.5 m (c) 187.5 m (d) 155.5 m

(c)

Distance travelled by first train, $s_1 = \frac{u^2}{2a} = \frac{(15)^2}{2} = 112.5\,m$

Distance travelled by second train, $s_2 = \frac{(20)^2}{2} = 200\,m$

Total distance travelled = 312.5m

Distance of separation = 500 – 312.5 = 187.5 m.

32. B_1, B_2 and B_3 are three balloons ascending with the velocities v, 2v and 3v respectively. If a bomb dropped from each when they are at the same height, then

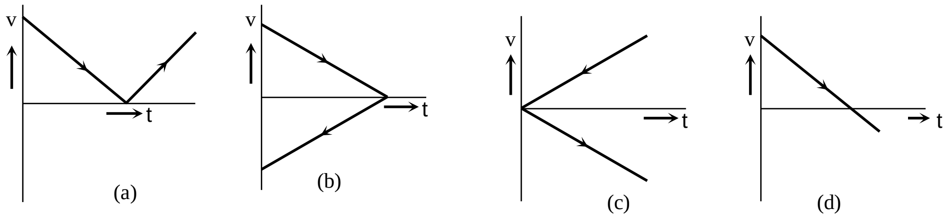
(a) Bomb from B_1 reaches ground first (b) Bomb from B_2 reaches ground first

(c) Bomb from B_3 reaches ground first (d) They reach ground simultaneously.

(a)

Bomb B_1 will have less velocity upwards on dropping, so it will reach ground first.

33. A ball is thrown vertically upwards. Which of the following graph/graphs represent velocity-time graph of the ball during its flight (air resistance is neglected)



(d)

In the positive region the velocity decreases linearly (during rise) and in the negative region velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

34. An object moving with a speed of 6.25 ms^{-1}, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(a) 1 s (b) 2 s (c) 4 s (d) 8 s

(b)

$\frac{dv}{dt} = -2.5\sqrt{v}$ Or $\frac{1}{\sqrt{v}}dv = -2.5\,dt$

On integrating, within limit ($v_1 = 6.25\ ms^{-1}$ to $v_2 = 0$)

$\therefore \int_{6.25}^{0} v^{-1/2}dv = -2.5\int_{0}^{t} dt$

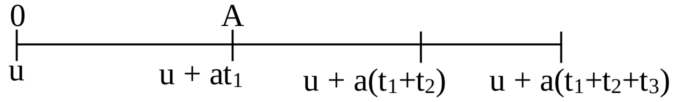
$2\left[v^{1/2}\right]_{6.25}^{0} = -(2.5)t$

Or $t = \frac{-2 \times (6.25)^{1/2}}{-2.5} = 2\ s$

35. A particle moving with uniform acceleration has average velocities v_1, v_2 and v_3 over the successive intervals of time t_1, t_2 and t_3 respectively. The value of $(v_1 - v_2) / (v_2 - v_3)$ will be:

(a) $\frac{(t_1 - t_2)}{(t_2 - t_3)}$ (b) $\frac{(t_1 - t_2)}{(t_2 + t_3)}$ (c) $\frac{(t_1 + t_2)}{(t_2 - t_3)}$ (d) $\frac{(t_1 + t_2)}{(t_2 + t_3)}$

(d)



Average velocities in intervals from 0 to t_1, t_1 to t_2 and t_2 to t_3 are:

$$v_1 = \frac{u + u + at_1}{2} = u + \frac{a}{2}t_1 \qquad \text{using, } v_{av} = \frac{u+v}{2}$$

$$v_2 = \frac{u + at_1 + u + a(t_1 + t_2)}{2} = u + at_1 + \frac{a}{2}t_2$$

$$v_3 = \frac{u + a(t_1 + t_2) + u + a(t_1 + t_2 + t_3)}{2} = u + at_1 + at_2 + \frac{a}{2}t_3$$

$$v_2 - v_1 = \frac{a}{2}(t_1 + t_2)$$

$$v_3 - v_2 = \frac{a}{2}(t_2 + t_3)$$

$$\therefore \frac{v_2 - v_1}{v_3 - v_2} = \frac{(t_1 + t_2)}{(t_2 + t_3)} \text{ Or } \frac{v_1 - v_2}{v_2 - v_3} = \frac{(t_1 + t_2)}{(t_2 + t_3)}$$

36. The relation between time t and distance x is: $t = \alpha x^2 + \beta x$, where α and β are constants. The retardation is

(a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^3$

(a)

$t = \alpha x^2 + \beta x = x(\alpha x + \beta)$

$$1 = \alpha(2x)\frac{dx}{dt} + \beta\frac{dx}{dt} \quad \therefore \quad v = \frac{dx}{dt} = \frac{1}{\beta + 2\alpha x}$$

$$a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{-2\alpha v}{(\beta + 2\alpha x)^2} = 2\alpha v^3$$

37. A train has a speed of 60 km/h, for the first one hour and 40 km/h for the next half an hour. Its average speed in km/h is

(a) 50 (b) 53. 33 (c) 48 (d) 70

(b)

Distance travelled by train in first 1 hour is 60 km and distance in next ½ hour is 20 km.

So Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{60 + 20}{3/2} = 53.33\,\text{km/h}$

38. The acceleration a of a particle starting from rest varies with time according to relation $a = \alpha t + \beta$. The velocity of the particle after a time t will be

(a) $\frac{\alpha t^2}{2} + \beta$ (b) $\frac{\alpha t^2}{2} + \beta t$ (c) $\alpha t^2 + \frac{1}{2}\beta t$ (d) $\frac{(\alpha t^2 + \beta)}{2}$

(b)

According to given relation acceleration $a = \alpha t + \beta$

As $a = \frac{dv}{dt} = \alpha t + \beta$

Since particle starts from rest, its initial velocity is zero i.e., at time t = 0, velocity = 0. $\Rightarrow \int_0^v dv = \int_0^t (\alpha t + \beta)dt \Rightarrow v = \frac{\alpha t^2}{2} + \beta t$

39. A point initially at rest moves along x-axis. Its acceleration varies with time as a = (6t + 5) m/s^2. If its starts from origin, the distance covered in 2 s is
(a) 20 m (b) 18 m (c) 16m (d) 25 m

(b)
$a = \frac{dv}{dt} = 6t + 5$
Or dv = (6t + 5)dt
Integrating it, we have; $\int_0^v dv = \int_0^t (6t+5)dt$
$\therefore\ v = \frac{6t^2}{2} + 5t + C$ (where C is constant of integration)
When t = 0, v = 0, so C = 0
$\therefore\ v = \frac{ds}{dt} = 3t^2 + 5t$
Or ds = (3t^2 + 5t)dt
Integrating it within the condition of motion, i.e., as t changes from 0 to 2s, s changes from 0 to s, we have;
$\int_0^s ds = \int_0^2 (3t^2 + 5t)dt$
$\therefore\ s = \left[t^3 + \frac{5}{2}t^2\right]_0^2 = 8 + 10 = 18\,m$

40. The velocity time relation of an electron starting from rest is given by v = 2t (in ms^{-1}). The distance travelled in 3 s is
(a) 9 m (b) 16 m (c) 27 m (d) 36 m

(a)
Given that, v = 2t
Since u = 0, v = at ⇒ a = 2 ms^{-2}
As the electron starts from rest, $S = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 3^2 = 9m.$

41. A body is moving with uniform acceleration describes 40 m in the first 5 s and 65 m in next 5 s. Its initial velocity will be
(a) 4 m/s (b) 2.5 m/s (c) 5.5 m/s (d) 11 m/s

(c)
$a = \frac{s_2 - s_1}{t^2} = \frac{65 - 40}{25} = 1ms^{-2}$
$s = ut + \frac{1}{2}at^2$
40 = 5u + ½ × 1 × 25
40 – 12.5 = 5u
5u = 27.5
u = 5.5 ms^{-1}

42. Two bodies of different masses m_a and m_b are dropped from two different heights a and b. The ratios of the time taken by the two bodies to cover these distances are

(a) a : b (b) b : a (c) $\sqrt{a} : \sqrt{b}$ (d) $a^2 : b^2$

(c)

$$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{2h/g}$$

$$t_a = \sqrt{\frac{2a}{g}} \text{ and } t_b = \sqrt{\frac{2b}{g}} \Rightarrow \frac{t_a}{t_b} = \sqrt{\frac{a}{b}}$$

43. Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?

(a) 2.50 m (b) 3.75 m (c) 4.00 m (d) 1.25 m

(b)

Time taken by first drop to reach the ground t = $\sqrt{\frac{2h}{g}}$

$$\Rightarrow t = \sqrt{\frac{2 \times 5}{10}} = 1s$$

As the water drop fall at regular intervals from a tap therefore time difference between any two drops = 0.5 s. Therefore, distance travelled by the second drop in 0.5s is, s=$\frac{1}{2g}\left(\frac{1}{2}\right)^2 = \frac{10}{8} = 1.25m$

Its distance from the ground = 5 - 1.25 = 3.75m

44. A balloon is rising vertically up with a velocity of 29 ms^{-1}. A stone is dropped from it and it reaches the ground in 10 s. The height of the balloon when the stone was dropped from it is (g = 9.8 ms^{-2})

(a) 100 m (b) 200 m (c) 400 m (d) 150 m

(b)

Since the stone is dropped from a moving balloon its initial velocity is 29 m/s downward.

u = - 29 m/s, t = 10 s

h = ut + ½ at^2

h = -29 × 10 $\frac{1}{2} \times 9.8 \times 100$

= - 290 + 490 = 200 m.

45. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(a) $v_0 t + \frac{1}{3}bt^2$ (b) $v_0 t + \frac{1}{3}bt^3$ (c) $v_0 t + \frac{1}{6}bt^3$ (d) $v_0 t + \frac{1}{2}bt^2$

(c)

$$\frac{dv}{dt} = bt \Rightarrow dv = bt\,dt \Rightarrow v = \frac{bt^2}{2} + K_1$$

At t=0, $v=v_0 \Rightarrow K_1=v_0$

We get v = $\frac{1}{2}bt^2 + v_0$

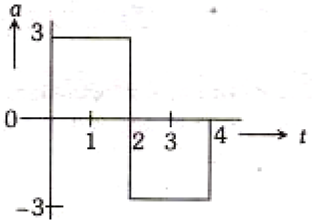
Again $\frac{dx}{dt} = \frac{1}{2}bt^2 + v_0$

$$\Rightarrow x = \frac{1}{2}\frac{bt^3}{3} + v_0 t + K_2$$

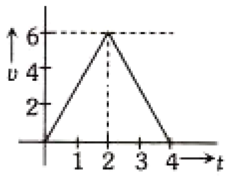
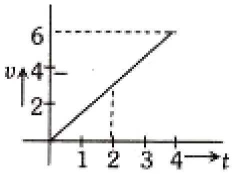
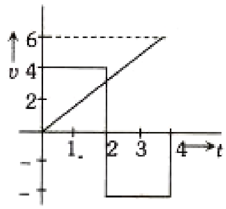
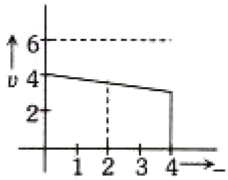
At t = 0, x = 0 $\Rightarrow K_2 = 0$

$$\therefore x = \frac{1}{6}bt^3 + v_0 t$$

46. A particle starts from rest at $t = 0$ and undergoes an acceleration a in ms^{-2} with time $'t'$ in seconds which is as shown.



Which one of the following plot represents velocity V in ms^{-1} versus time $'t'$ in seconds.

(a)  (b)  (c)  (d) 

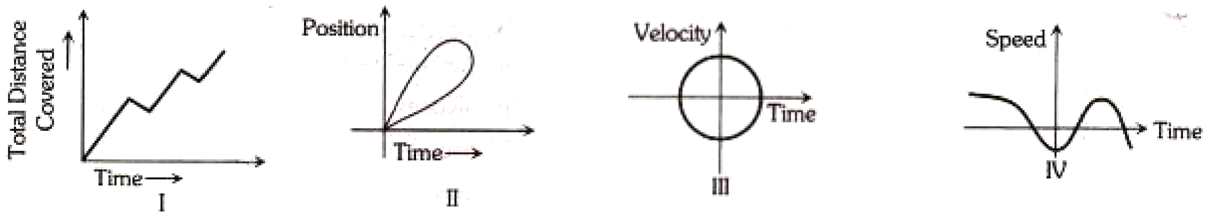
(a)

Taking the motion from 0 to 2 s, u=0, a=3ms^{-2}, t=2s, v=?

v = u + at = 0+3×2 = 6 ms^{-1}.

Taking the motion from 2s to 4s, v = 6+(-3)(2) = 0

47. Which of the following graphs cannot possibly represent one dimensional motion of a particle?



(a) I and II (b) II and III (c) II and IV (d) All four

(d)

I is not possible because total distance covered by a particle increases with time.

II is not possible because at a particular time, position cannot have two values.

III is not possible because at a particular time, velocity cannot have two values.

IV is not possible because speed can never be negative.

48. Two identical balls are shot upward one after another at an interval of 2s along the same vertical line with same initial velocity of 40 ms^{-1}. The height at which the balls collide is

(a) 50 m (b) 75 m (c) 100 m (d) 125 m

(b)

$s_1 = s_2$

$\therefore (40)t = \frac{1}{2}\times 10\times t^2 = (40)(t-2)$

$-\frac{1}{2}\times 10\times (t-2)^2$

Solving this equation, we get t = 5 s

Then $s_1 = (40)(5) - \frac{1}{2}\times 10\times (5)^2$ =75 m

49. The velocity υ and displacement r of a body are related as υ^2 = kr, where k is a constant. What will be the velocity after 1 second? (Given that the displacement is zero at t = 0)

(a) $\sqrt{kr}$ (b) $kr^{3/2}$

(c) $\frac{k}{2}r^0$ (d) Data is not sufficient

(c)

υ^2 = kr or $\upsilon = \sqrt{kr}$

$a = \frac{d\upsilon}{dt} = \sqrt{k}\,\frac{1}{2}r^{-1/2}\frac{dr}{dt} = \sqrt{k}\,\frac{1}{2}r^{-1/2}.\upsilon = \sqrt{k}\,\frac{1}{2}r^{-1/2}.\sqrt{k}r^{-1/2} = \frac{k}{2}r^0$

Velocity after 1 s is v = u +at = 0 + $\frac{k}{2}r^0 \times 1 = \frac{k}{2}r^0$

50. A stone is dropped from the 25th storey of a multi-storeyed building and it reaches the ground in 5s. In the first second, it passes through how many storeys of the building (g= 10 ms^{-2})

(a) 1 (b) 2 (c) 3 (d) None

(a)

Suppose h be the height of each storey, then

$25h = 0 + \frac{1}{2}\times 10\times t^2 = \frac{1}{2}\times 10\times 5^2$ or $h = 5m$

In first second, let the stone passes through n storey.

So $n\times 5 = \frac{1}{2}\times 10\times (1)^2$ or $n = 1$