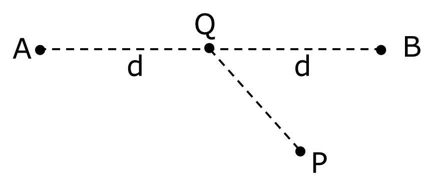
2PUC – CHAPTER 02
ELECTRIC POTENTIAL AND CAPACITANCES

1. Identify the **wrong** statement.
(a) The electrical potential energy of a system of two protons shall increase if the separation between the two is decreased.
(b) The electrical potential energy of a proton, electron system will increase if the separation between the two is decreased.
(c) The electrical potential energy of a proton, electron system will increase if the separation between the two is increased.
(d) The electrical potential energy of system of two electrons increase if the separation between the two is decreased.
(b)
The potential energy is relative quantity, it depends on reference position. While comparing potential energy in two position, consider negative sign i.e., if $U_1 = -2J$, $U_2 = -3J$, $U_3 = 0$, $U_3 > U_1 > U_2$
(a)$U \propto \frac{e^2}{r}$, if ‘r’ is decreased then U increases
(b) $U \propto \frac{-e^2}{r}$, if ‘r’ is decreased then U decreases
(c) $U \propto \frac{-e^2}{r}$, if ‘r’ is increased then U increases
(d) $U \propto \frac{e^2}{r}$, if ‘r’ is decreased then U increases

2. The work done in taking a unit positive charge from P to A is W_A and from P to B is W_B. Then



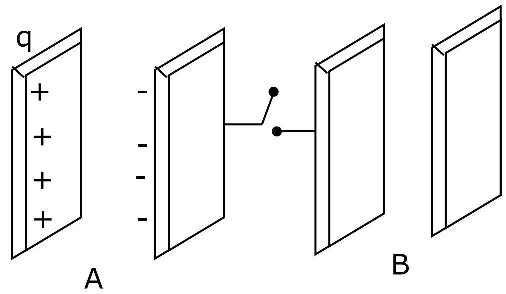
(a) $W_A > W_B$
(b) $W_A < W_B$
(c) $W_A = W_B$
(d) $W_A + W_B = 0$

(c)
As potential at A and B is the same, $V_A = V_B = kQ/d$.
Potential difference = $V_{AP} = V_{AB}$
W = Charge moved × electric potential difference
So work done in both the cases will be the same.

3. Mark the **correct** statement.
(a) If E is zero at a certain point, then V should be zero at that point.
(b) If E is not zero at a certain point, then V should not be zero at that point.
(c) If V is zero at a certain point, then E should be zero at that point.
(d) If V is zero at a certain point, then E may or may not be zero.
(d)
E = - (dV/dx)

4. Electric potential is given by $V = 6x - 8y^2 - 8y + 6yz - 4z^2$. Then electric force acting on 2C point charge placed origin will be
(a) 2 N (b) 6 N (c) 8 N (d) 20 N
(d)
$E_x = -\frac{dV}{dx} = -(6 - 8y^2)$, $E_y = -\frac{dV}{dy} = -(-16xy - 8 + 6z)$, $E_z = -\frac{dV}{dz} = -(6y - 8z)$
At origin $x = y = z = 0$ So, $E_x = -6$, $E_y = 8$ and $E_z = 0$ $\Rightarrow E = \sqrt{E_x^2 + E_y^2} = 10\,NC^{-1}$
Hence force F = qE = $2 \times 10 = 20\,N$.

5. Consider the situation shown in the figure. The capacitor A has a charge 'q' on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(a) zero (b) q/2 (c) q (d) 2q

(a)

The $\pm q$ charges appearing on the inner surface of A are bounded charges. As B is uncharged initially, as it isolated the charges on A will not be affected on closing the switch S. No charge will flow into B.

6. The plates of a parallel plate capacitor are pulled apart with a velocity v. If at any instant their mutual distance of separation is x, then magnitude of rate of change of capacitance with respect to time varies as

(a) 1/x (b) $1/x^2$ (c) x^2 (d) x

(b)

$$C = \frac{A\in_0}{x} = A\in_0 x^{-1}$$

$$\frac{dC}{dt} = A\in_0 (-1)x^{-2}\frac{dx}{dt} = -\frac{A\in_0}{x^2}v$$

$$\left|\frac{dC}{dt}\right| \propto \frac{1}{x^2}.$$

7. The potential function of an electrostatic field is given by $V = 2x^2$. Determine the electric field strength at the point (2m, 0, 3m).

(a) $\vec{E} = 4\hat{i}(NC^{-1})$ (b) $\vec{E} = -4\hat{i}(NC^{-1})$ (c) $\vec{E} = 8\hat{i}(NC^{-1})$ (d) $\vec{E} = -8\hat{i}(NC^{-1})$

(d)

$$E_x = -\frac{dV}{dx} = -4x = -4\times 2 = -8$$

$$E_y = 0, E_z = 0$$

Hence, $\vec{E} = -8\hat{i}\ NC^{-1}$.

8. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is the same for the both spheres, the electric potential at the common centre will be

(a) $\frac{\sigma}{\varepsilon_0}\cdot\frac{r_1}{r_2}$ (b) $\frac{\sigma}{\varepsilon_0}\cdot\frac{r_2}{r_1}$ (c) $\frac{\sigma}{\varepsilon_0}(r_1 - r_2)$ (d) $\frac{\sigma}{\varepsilon_0}(r_1 + r_2)$.

(d)

The electric potential at the common centre is $V = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$

Now, $\sigma = \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}$

$$\therefore V = \frac{1}{\varepsilon_0}\left[\frac{q_1 r_1}{4\pi r_1^2} + \frac{q_2 r_2}{4\pi r_2^2}\right] = \frac{\sigma}{\varepsilon_0}(r_1 + r_2).$$

9. A soap bubble of radius 'r' is charged to a potential V. If the radius is increased to 'nr', the potential on the bubble will become

(a) nV (b) n^2V (c) $\frac{V}{n}$ (d) $\frac{V}{n^2}$

(c)

If the radius of a bubble is increased by a factor n, its capacitance is also increased by a factor n. i.e., $C' = nC$. Since the charge Q on the bubble remains unchanged, we have

$$Q = CV = C'V' \text{ or } V' = \frac{CV}{C'} = \frac{CV}{nC} = \frac{V}{n}.$$

10. A metallic sphere A of radius ‘a’ carries a charge Q. It is brought in contact with an uncharged sphere B of radius ‘b’. The charge on sphere A now will be

(a) $\frac{aQ}{b}$ (b) $\frac{bQ}{a}$ (c) $\frac{bQ}{a+b}$ (d) $\frac{aQ}{a+b}$

(d)

Charge will flow from A to B until their potentials become equal.

If charge ‘q’ flows from A to B, then $\frac{Q-q}{4\pi\varepsilon_0 a} = \frac{q}{4\pi\varepsilon_0 b}$

or $Q-q = \frac{a}{b}q$ which gives $q = \frac{bQ}{a+b}$.

Hence charge left on $A = Q-q = Q-\frac{bQ}{a+b} = \frac{aQ}{a+b}$.

11. A metallic sphere of radius R is charged to a potential V. The magnitude of the electric field at a distance r(> R) from the centre of the sphere is

(a) $\frac{V}{r}$ (b) $\frac{Vr}{R^2}$ (c) $\frac{VR}{r^2}$ (d) zero

(c)

Let the charge on the sphere be Q. Then $V = \frac{Q}{4\pi\varepsilon_0 R}$ which gives $Q = 4\pi\varepsilon_0 RV$

The electric field at a distance ‘r’ is $E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{4\pi\varepsilon_0 RV}{4\pi\varepsilon_0 r^2} = \frac{RV}{r^2}$.

12. Two concentre thin metallic spheres of radii R_1 and R_2 ($R_1 > R_2$) bear charges Q_1 and Q_2 respectively. Then potential at radius ‘r’ between R_1 and R_2 will be

(a) $k\left(\frac{Q_1+Q_2}{r}\right)$ (b) $k\left(\frac{Q_1}{r}+\frac{Q_2}{R_2}\right)$ (c) $k\left(\frac{Q_2}{r}+\frac{Q_1}{r}\right)$ (d) $k\left(\frac{Q_1}{R_1}+\frac{Q_2}{r}\right)$

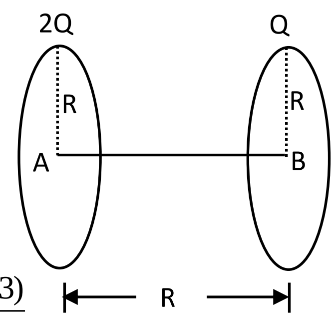
(d)

For inner sphere the point lies outside.

For outer sphere the point lies inside.

$$V_P = \frac{1}{4\pi\in_0}\left[\frac{Q_2}{r}+\frac{Q_1}{R_1}\right] = k\left[\frac{Q_1}{R_1}+\frac{Q_2}{r}\right].$$

13. In the following arrangement



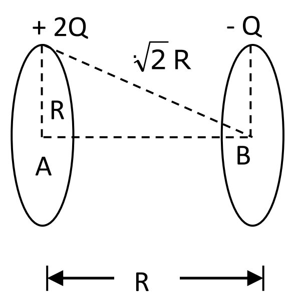
(a) $V_A = \frac{Q(2\sqrt{2}-1)}{4\pi\in_0\sqrt{2}R}$

(b) $V_B = \frac{Q(\sqrt{2}-1)}{4\pi\in_0 R}$

(c) Work done in moving a charge + q from B to A, $W_{BA} = \frac{qQ(3\sqrt{2}-3)}{4\sqrt{2}\pi\in_0 R}$

(d) all the options are correct.

(d)



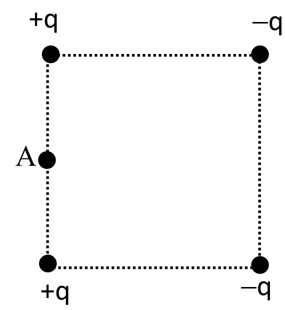
$$V_A = \frac{1}{4\pi\in_0}\left[\frac{2Q}{R}+\frac{-Q}{\sqrt{2}R}\right] = \frac{Q(2\sqrt{2}-1)}{4\pi\in_0\sqrt{2}R}$$

$$V_B = \frac{1}{4\pi\in_0}\left[\frac{-Q}{R}+\frac{2Q}{\sqrt{2}R}\right] = \frac{Q}{4\pi\in_0 R}[\sqrt{2}-1]$$

$$= \frac{Q}{4\pi\in_0\sqrt{2}R}[(2-\sqrt{2})]$$

$$W_{B\to A} = q(V_A - V_B) = \frac{qQ}{4\sqrt{2}\pi\in_0 R}[3\sqrt{2}-3]$$

14. Four electric charges $+q, +q, -q$ and $-q$ are placed at the corners of a square of side 2L (see figure). The electric potential at point A, midway between the two charges $+q$



(a) zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}(1+\sqrt{5})$

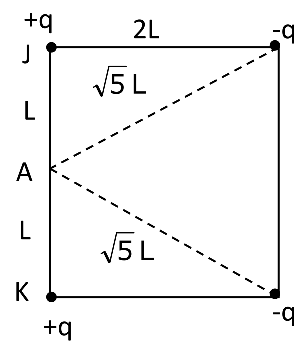
(c) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$

(d)

$$V_A = \frac{1}{4\pi \in_0}\left[\frac{q}{L}\times 2 + \frac{-q}{\sqrt{5}L}\times 2\right]$$

$$= \frac{1}{4\pi \in_0}\frac{2q}{L}\left[1-\frac{1}{\sqrt{5}}\right].$$



15. Charges 2q and 8q are placed at the end points A and B respectively of a 9 cm long straight line. A third charge 'q' is placed at a point C of AB such that the potential energy of the system is minimum. The distance of C from A is

(a) 2 cm (b) 3 cm (c) 4cm (d) 5 cm

(b)

Let x be the distance between the charge at A and C

$$U = \frac{1}{4\pi \in_0}\left[\frac{2q\cdot q}{x} + \frac{q\cdot 8q}{(9-x)} + \frac{q.8q}{9}\right]$$

For U to be minimum: $\frac{dU}{dx} = 0 \Rightarrow -\frac{2}{x^2} + \frac{8}{(9-x)^2} = 0$

$9 - x = 2x \Rightarrow x = 3cm$.

16. A conducting sphere of radius 10 cm is charged to $10\,\mu C$. Another uncharged sphere of radius 20 cm is allowed to touch it for some time. After that if the spheres are separated, then surface density of charges, on the spheres will be in the ratio of

(a) 1 : 4 (b) 1 : 3 (c) 2 : 1 (d) 1 : 1

(c)

When two spheres are placed in contact, they attain same potential.

$$\sigma = \frac{E}{\varepsilon_0} = \frac{V}{R\varepsilon_0}$$

For same V, $\sigma \propto \frac{1}{R}$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = \frac{2}{1}.$$

17. If identical charges $(-q)$ are placed at each corner of a cube of side b, then electric potential energy of charge $(+q)$ which is placed at centre of the cube will be

(a) $\frac{8\sqrt{2}q^2}{4\pi\varepsilon_0 b}$ (b) $\frac{-8\sqrt{2}q^2}{\pi\varepsilon_0 b}$ (c) $\frac{-4\sqrt{2}q^2}{\pi\varepsilon_0 b}$ (d) $\frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 b}$

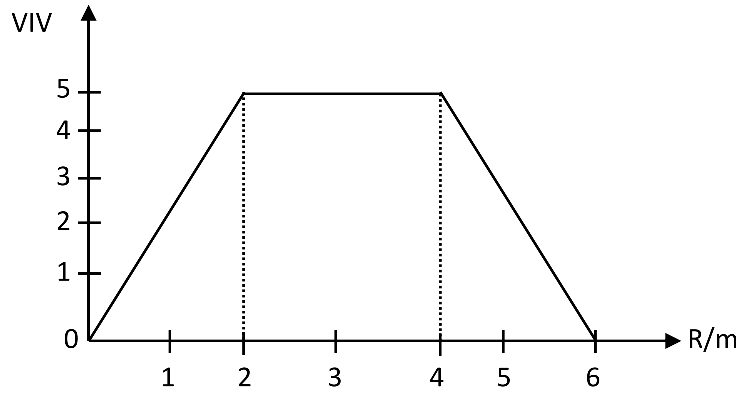
(d)

Number of pairs = 8

$$U = \frac{1}{4\pi \in_0}\cdot\frac{-q\cdot q}{(\sqrt{3}b/2)}\times 8 = \frac{-4q^2}{\sqrt{3}\pi \in_0 b}.$$

18. The variation of potential with distance R from the fixed point is shown in figure.

The electric field at R = 5 m is



(a) $2.5\ Vm^{-1}$ (b) $-2.5\ Vm^{-1}$

(c) $0.4\ Vm^{-1}$ (d) $-0.4\ Vm^{-1}$

(a)

$E = -\frac{dV}{dr}$ = negative of the slope of V-r graph.

$E = -\frac{0-5}{6-4} = 2.5V/m$

19. The potential V is varying with 'x' as $V = \frac{1}{2}(y^2 - 4x)$ volt. The field at $x = 1m$, $y = 1m$ is

(a) $2\hat{i} + \hat{j}\ Vm^{-1}$ (b) $-2\hat{i} + \hat{j}\ Vm^{-1}$ (c) $2\hat{i} - \hat{j}\ Vm^{-1}$ (d) $-2\hat{i} + 2\hat{j}\ Vm^{-1}$

(c)

$E_x = -\frac{dV}{dx} = -\frac{1}{2}[-4] = 2$

$E_y = -\frac{dV}{dy} = -\frac{1}{2}[2y] = -y = -1$

$\therefore\ \vec{E} = E_x\hat{i} + E_y\hat{j}$

20. The concentric spheres of radii R and 2R are charged. The inner sphere has a charge of 1μC and the outer sphere has 2μC of the same sign. The potential is 9000 V at a distance 3 R from the common centre. What is value of R?

a) 1m b) 2m c) 3m d) 4m

(a)

For the purpose of calculations, the charges on the spheres may be considered to reside at the centre of the spheres. The total charge is supposed to reside at the common centre of the concentric spheres.

q = 1μC + 2μC = 3μC

Potential at a distance 3R from the common centre V = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{3r}$

or 9000 = $\frac{9 \times 10^9 \times 3 \times 10^{-6}}{3R}$

i.e. , R = 1m

21. Three point charges q, 2q and 8q are to be placed on a 9 cm long straight line. Find the position of the charge q such that potential energy of this system minimum.

a) 1 cm from 2 q b) 2 cm from 2q c) 3 cm from 2 q d) 4 cm from 2q

(c)

As potential energy of two point charges U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$

So to have minimum potential energy, the charges of greater values should be farthest

i.e. q must be between 2q and 8q.

Let q be at a distance x from 2q, the potential energy of the system will be

U = $\frac{1}{4\pi\varepsilon_0}\left[\frac{2qq}{x} + \frac{8qq}{(d-x)} + \frac{8q+2q}{d}\right]$

For U to be minimum (dU/dx) = 0

i.e., 2x = d – x

or x = d/3 = 3 cm

22. Three concentric spherical shells have radii a, b and c (a < b < c) and have surface charge densities σ, -σ and σ, respectively. If V_A, V_B and V_C denote the potentials of the three shell, then for c = a +b, we have

a) $V_C \neq V_B \neq V_A$ b) $V_C = V_A \neq V_B$ c) $V_C = V_B = V_A$ d) $V_C \neq V_B = V_A$

(b)

We have, $V_A = \frac{\sigma}{\varepsilon_0}(a - b + c)$

$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b} - b + c\right)$ and $V_C = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c} - \frac{b^2}{c} + c\right)$

Putting these, c = a + b $\Rightarrow V_A = V_C \neq V_B$ or $V_C = V_A \neq V_B$

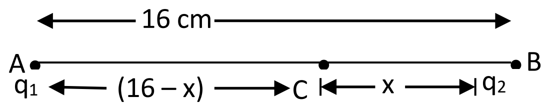
23. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

a) 6 cm from the charge -3×10^{-8} C b) 6 cm from the charge 5×10^{-8} C

c) 9 cm from the charge -3×10^{-8} C d) 9 cm from the charge 5×10^{-8} C

(c)

Let the potential be zero at point C at a distance x from point B. Let us consider two charges q_1 and q_2. According to the question,



$q_1 = 5 \times 10^{-8}C$, $q_2 = -3 \times 10^{-8}C$, AB = 16cm = 16×10^{-2} m

The potential at point C due to charge q_1 is $V_A = \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{AC} = \frac{9\times10^{-9}\times5\times10^{-8}}{(16-x)\times10^{-2}}$ ...(i)

The potential at point C due to charge q_2 is $V_B = \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{RC} = \frac{-9\times10^{9}\times3\times10^{-8}}{(16-x)\times10^{-2}}$ ...(ii)

Now the net potential at point C is zero i.e., $V_A + V_B = 0$

Putting the values from Eqs. (i) and (ii), we get

$$\frac{9\times10^{9}\times5\times10^{-8}}{(16-x)\times10^{-2}} + \left(\frac{-9\times10^{9}\times3\times10^{-8}}{x\times10^{-2}}\right) = 0$$

Or $\frac{5}{16-x} - \frac{3}{x} = 0$ or 5x – 3(16 – x) = 0 Or 5x – 48 + 3x = 0

or 8x = 48

x = 6cm

Thus, the electric potential is zero at the distance of 6 cm from q_2 $(-3 \times 10^{-8}$ C)

24. Two insulated metal spheres of radii 10 cm and 15 cm charged to a potential of 150 V and 100 V respectively, are connected by means of a metallic wire. What is the charge on the first sphere?

a) 1 nC b) 1.333 nC c) 1.5 nC d) 0.75 nC

(b)

Here, r_1 = 10cm, r_2 = 15cm, V_1 = 150 V, V_2 = 100 V

Common potential, $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{4\pi\varepsilon_0(r_1V_1 + r_2V_2)}{4\pi\varepsilon_0(r_1 + r_2)}$ = 120 V

$\therefore q_1 = C_1V = 4\pi\varepsilon_0 r_1 V = \frac{10^{-1}}{9\times10^{9}}\times120\,C = \frac{12}{9\times10^{9}}$ = 1.333 nC

25. A charge (-q) and another charge (+Q) are kept at two points A and B, respectively. Keeping the charge (+Q) fixed at B, the charge (-q) at A is moved to another point C such that ABC forms an equilateral triangle of side ℓ. The net work done in moving the charge (-q) is

a) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\ell}$ b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}$ c) $\frac{1}{4\pi\varepsilon_0}Qq\ell$ d) zero

(d)

As, net work done = final PE – initial PE = $\frac{Qq}{4\pi\varepsilon_0\ell} - \frac{Qq}{4\pi\varepsilon_0\ell} = \text{zero}$

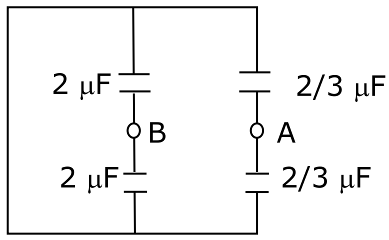
26. The equivalent capacitance of the circuit across the terminals A and B is equal to

(a) $0.5\mu F$

(b) $2\mu F$

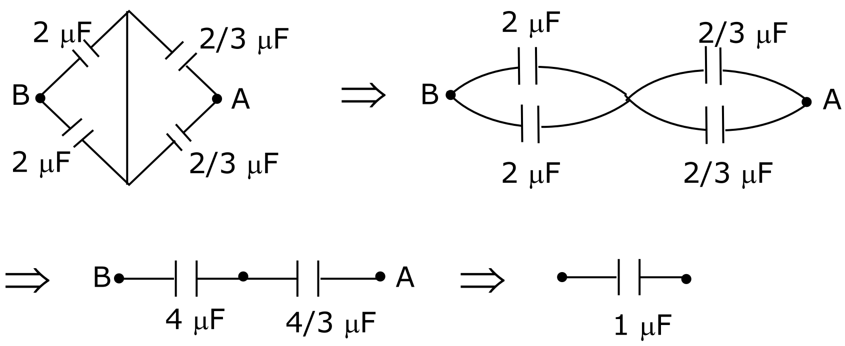
(c) $1\mu F$

(d) none of these



(c)

The redrawn circuit is as follows:



27. An electric field is given by E = $(y\hat{i} + x\hat{i})$ NC^{-1}. The work done in moving a 1 C charge from $\vec{r}_A = (2\hat{i} + 2\hat{j})$ m to $\vec{r}_B = (4\hat{i} + 2\hat{j})$m is

a) 2 y   b) 3 y   c) zero   d) infinity

(a)

Here, displacement would be, $\Delta\vec{r} = (4-2)\hat{i} + (2-2)\hat{j} = 2\hat{i}$

Now, $\Delta V = \vec{E}.\Delta\vec{r} = (y\hat{i} + x\hat{j}).2\hat{i} = 2y$

$\therefore$ Work done, W = q(ΔV) = q. 2y = 1 × 2y = 2y J   ($\because$ charge = 1C)

28. A capacitor of capacitance $C_1 = 1\mu F$ can withstand maximum voltage $V_1 = 6kV$ and another capacitor of capacitance $C_2 = 3\mu F$ can withstand maximum voltage $V_2 = 4kV$. When the two capacitors are connected in series, the combined system can withstand a maximum voltage of

(a) 4 kV   (b) 6 kV   (c) 8 kV   (d) 10 kV

(c)

$Q_1 = C_1V_1 = 1 \times 6 = 6$ mC

$Q_2 = C_2V_2 = 3 \times 4 = 12$ mC

$$V = \frac{Q_{min}}{C_s} = \frac{6\times10^{-3}}{\frac{3\times1}{3+1}\times10^{-6}} = 8kV$$

29. A sphere of radius r is charged to a potential V. The outward pull per unit area of its surface is given by

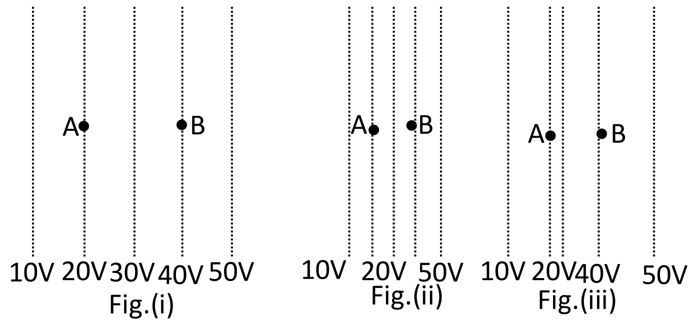
a) $\frac{4\pi\varepsilon_0 V^2}{r^2}$ b) $\frac{\varepsilon_0 V^2}{2r^2}$ c) $\frac{2\pi\varepsilon_0 V^2}{r^2}$ d) $\frac{\varepsilon_0 V^2}{4r^2}$

(b)

Electrical pressure (force/area) p = $\frac{1}{2}\varepsilon_0 E^2$

$\therefore$ p = $\frac{1}{2}\varepsilon_0 \frac{V^2}{r^2}$ $\because$ E = $\frac{V}{r}$

30. Figures shows some equipotential lined distributed in space. A charged object is moved from point A to point B.



a) The work done in Fig. (i) is the greatest

b) The work done in Fig. (ii) is least

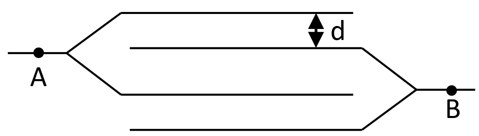
c) The work done in Fig. (i), Fig. (ii) and Fig. (iii) is same

d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in Fig. (i)

(c)

We observe that in all the three parts, $V_A = 20$ V and $V_B = 40$ V. Work done in carrying a charge q from A to B is $W = q(V_B - V_A)$ = same in all the three figures

31. The equivalent capacity between points A and B in figure will be, while capacitance of each capacitor is 3μF.



a) 2 μF b) 4 μF c) 7 μF d) 9 μF

(d)

Positive plate of all the three condensers is connected to one point A and negative plate of all the three condensers is connected to point B, i.e., they are joined in parallel.

$\therefore C_p = 3 + 3 + 3 = 9\mu F$

32. The electric potential at a point (x,y) in the xy- plane is given by V = -Kxy

The electric field intensity at a distance r from the origin varies as

a) r^2 b) r c) 2r d) $2r^2$

(b)

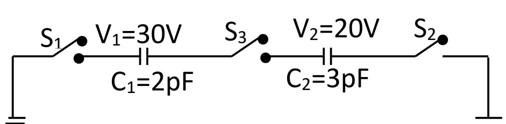
Distance of point from origin, $r = \sqrt{x^2 + y^2}$ and given V = -kxy

$\Rightarrow E_x = -\frac{dV}{dx} = \frac{d}{dx}(-kxy) = ky$ and $E_y = -\frac{dV}{dy}(-kxy) = kx$

$\therefore E = \sqrt{E_x^2 + E_y^2} = k\sqrt{y^2 + x^2} = kr$

i.e., $E \propto r$

33. For the circuit shown figure, which of the following statements is true?



a) With S_1 closed, $V_1 = 15V$, $V_2 = 20V$ b) With S_3 closed, $V_1 = V_2$, $V_2 = 20V$

c) With S_1 and S_3 closed, $V_1 = V_2 = 0$ d) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

(d)

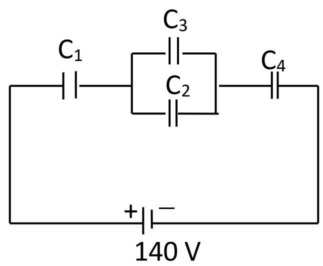
With S_1 and S_3 closed, the capacitors C_1 and C_2 are in series arrangement.

In series arrangement, potential difference developed across capacitors is in the inverse ratio of their capacities. Hence, $\frac{V_1'}{V_2'} = \frac{C_2}{C_1} = \frac{3\,pF}{2\,pF} = \frac{3}{2}$ and $V_1' + V_2' = V_1 + V_2 = 30 + 20 = 50$ V

On simplification, we get $V_1' = V_1 = 30$ V and $V_2' = V_2 = 20V$

34. In the circuit arrangement shown in figure, the value of $C_1 = C_2 = C_3 = 30$ pF and $C_4 = 120$ pF. If the combination of capacitors is charged with 140V DC supply, the potential differences across the four capacitors will be respectively

a) 80 V, 40V, 40V and 20V
b) 20 V, 40V, 40V and 80V
c) 35 V, 35V, 35V and 35V
d) 80 V, 20V, 20V and 20V



(a)

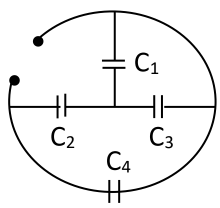
Here, $C_{23} = 30 + 30 = 60$ pF.

Total equivalent capacitance is given by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{30} + \frac{1}{60} + \frac{1}{120} = \frac{7}{120} \Rightarrow C = \frac{120}{7}$ pF

$\therefore$ Total charge, $Q = CV = \frac{120}{7} \times 140\text{ pC} = 2400\text{ pC}$ $\therefore V_1 = \frac{Q}{C_1} = \frac{2400\text{ pC}}{30\text{ pF}} = 80\text{ V}$

$V_2 = V_3 = V_{23} = \frac{Q}{C_{23}} = \frac{2400\text{ pC}}{30\text{ pF}} = 40\text{ V}$ and $V_4 = \frac{Q}{C_4} = \frac{2400\text{ pC}}{120\text{ pF}} = 20\text{ V}$

35. In the arrangement of capacitors shown in figure, each capacitor is 9 μF, then the equivalent capacitance between the points A and B is



a) 9 μF b) 18 μF c) 4.5 μF d) 15 μF

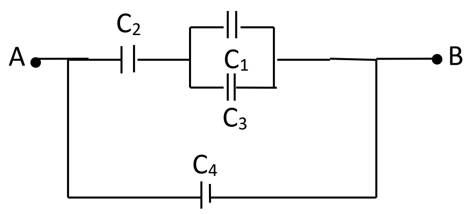
(d)

The arrangement can be redrawn as shown in the adjoining figure.

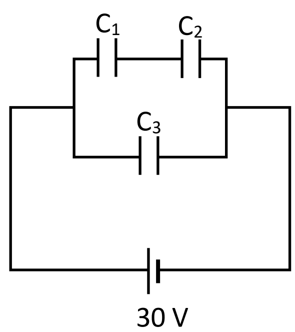
$\therefore C_{13} = C_1 + C_3 = 9 + 9 = 18\mu F$

and $C_{2\text{-}13} = \frac{C_2 \times C_{13}}{C_2 + C_{13}} = \frac{9\mu F \times 18\mu F}{(9+18)\mu F} = 6\mu F$

$\therefore C = C_{2\text{-}13} + C_4 = 6\mu F + 9\mu F = 15\mu F$



36. Two capacitors $C_1 = 3\mu F$ and $C_2 = 6\mu F$ in series, are connected in parallel to a third capacitor $C_3 = 4\mu F$. This arrangement is the connected to a battery of e.m.f = 30 V, as shown. The energy lost by the battery in charging the capacitors



(a) 900 μJ (b) 1800 μJ

(c) 2700 μJ (d) 3600 μJ

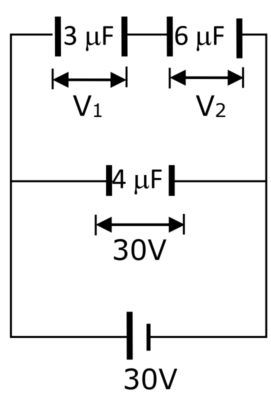
(c)

$V_1 = \left(\frac{6}{3+6}\right) \times 30 = 20V$

$V_2 = 30 - V_1 = 10V$

Heat produced $H = \left[\frac{1}{2} \times 3(20-0)^2 + \frac{1}{2} \times 6(10-0)^2 + \frac{1}{2} \times 4(30-0)^2\right] \times 10^{-6}$

= 2700 μJ



37. Two insulated metallic sphere of 3μF and 5μF capacitances are charged to 300 V and 500 V, respectively. The energy loss, when they are connected by a wire, is

a) 0.0375 J b) 0.235 J c) 0.375 J d) 375 J

(a)

We have, $\Delta U = \frac{1}{2}\frac{C_1C_2(V_2 - V_1)^2}{(C_1 + C_2)} = \frac{1}{2}\frac{(3 \times 5) \times 10^{-12} \times (500 - 300)^2}{(3+5) \times 10^{-6}}$

$= \frac{1}{2} \times \frac{15 \times 10^{-12} \times 4 \times 10^4}{8 \times 10^{-6}} = \frac{30 \times 10^{-2}}{8} \text{ J} = 0.0375 \text{ J}$

38. If on the concentric hollow sphere of radii r and R(>r) the charge Q is distributed such that their surface densities are same, then the potential at their common centre is

a) $\frac{Q(R^2 + r^2)}{4\pi\varepsilon_0(R+r)}$ b) $\frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$ c) zero d) $\frac{QR}{R+r}$

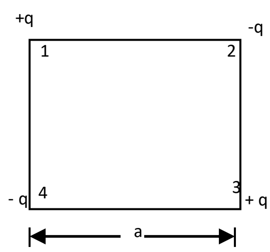
(b)

As, $q_1 + q_2 = Q$

Here, $\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$ $\therefore q_1 = \frac{Qr^2}{R^2 + r^2}$ and $q_2 = \frac{QR^2}{R^2 + r^2}$

$\therefore$ Potential at common centre $= \frac{1}{4\pi\varepsilon_0}\left(\frac{Qr^2}{(R^2+r^2)r} + \frac{QR^2}{(R^2+r^2)R}\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)}$

39. The work required to put the four charges at the corners of a square of side a, as shown in figure, is



a) $\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{a}$ b) $\frac{-2.6}{4\pi\varepsilon_0}.\frac{q^2}{a}$ c) $+\frac{2.6}{4\pi\varepsilon_0}.\frac{q^2}{a}$ d) $\frac{1}{4\pi\varepsilon_0}.\frac{4q^2}{a}$

(c)

As, work done = Potential energy of configuration of change

$= \frac{1}{4\pi\varepsilon_0 a}[q(-q) + (9 - q)q + q(-q) + (-q)(q)] = -\frac{(-q)(-q) + q^2}{4\pi\varepsilon_0 a\sqrt{2}}$

$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{4q^2}{a} + \frac{2q^2}{a\sqrt{2}}\right] = -\frac{2.6}{4\pi\varepsilon_0}\frac{q^2}{a}$

40. A parallel plate capacitor has the space between its plates filled by two slabs of thickness $\frac{d}{2}$ each and dielectric constants K_1 and K_2. d is the plate separation of the capacitor. The capacity of the capacitor is

a) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{K_1K_2}\right)$ b) $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$ c) $\frac{2\varepsilon_0 A}{d}(K_1 + K_2)$ d) $\frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{K_1K_2}\right)$

(b)

As, $C_1 = \frac{K_1\varepsilon_0 A}{d/2} = \frac{2K_1\varepsilon_0 A}{d}$

$C_2 = \frac{2K_1\varepsilon_0 A}{d}$

Now, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\varepsilon_0 A} + \frac{d}{2K_2\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right) \Rightarrow C_s = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1 + K_2}\right)$

41. Six identical capacitors are joined in parallel, charged to a potential difference of 10 V, separated and then connected in series, i.e., the positive plate of one is connected to negative plate of other. Then potential difference between free plates is

a) 10 V b) 30 V c) 60 V d) $\frac{10}{6}$ V

(b)

If C is capacity of each condenser, then charge on each capacitor = 10 C

When connected in series, p.d between free plates = $\frac{\text{totalcharge}}{\text{totalcapacity}} = \frac{10C}{C/6} = 60V$

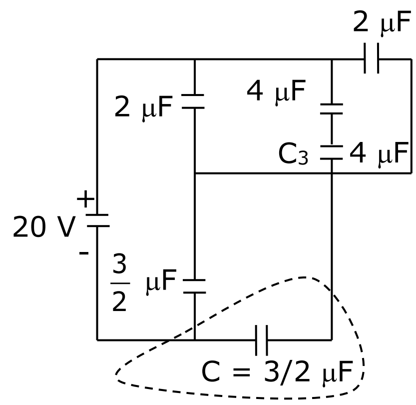
42. In figure, the battery has a potential difference of 20 V. The charge in the capacitor marked as C is

(a) 20 μC

(b) 40 μC

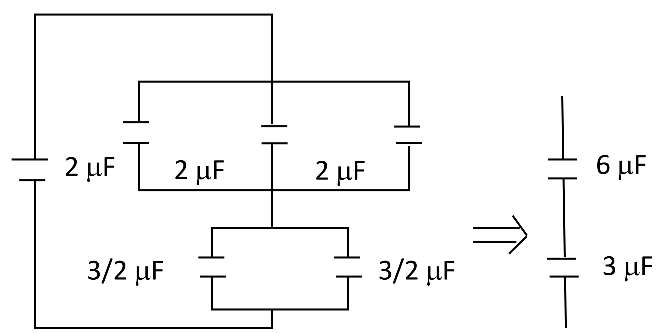
(c) 10 μC

(d) none of these



(a)

The circuit can be simplified as follows:



Hence $C_{eq} = 2\mu F$.

Thus, charge supplied by battery is $Q = 2\times 20 = 40\,\mu C$.

Charge on required capacitor is $20\,\mu C$.

43. The work done in placing a charge of $8\times10^{-18}C$ on a condenser of capacity $100\,\mu F$ is

(a) $16\times10^{-32}J$ (b) $3.1\times10^{-26}J$ (c) $4\times10^{-10}J$ (d) $32\times10^{-32}J$

(d)

Capacitance $C = 100\mu F = 100\times10^{-6}F = 10^{-4}F$

Charge Q = $8\times10^{-18}C$

Hence, the required work done is $W = \frac{1}{2}\frac{Q^2}{C^2} = \frac{1}{2}\times\frac{(8\times10^{-8})^2}{(10^{-4})} = 32\times10^{-32}J$.

44. The effective capacitance of two capacitors of capacitances C_1 and C_2 (with $C_2 > C_1$) connected in parallel is $\frac{25}{6}$ times the effective capacitance when they are connected in series. The ratio C_2/C_1 is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{25}{6}$

(a)

Given $C_1 + C_2 = \frac{C_1C_2}{C_1+C_2}\times\frac{25}{6}$ Or $6(C_1+C_2)^2 = 25C_1C_2$

Or $6C_1^2 + 6C_2^2 + 12C_1C_2 = 25C_1C_2$ Or $6C_1^2 + 6C_2^2 - 13C_1C_2 = 0$

Let $C_2 = xC_1$. Then, we have $6C_1^2 + 6x^2C_1^2 - 13xC_1^2 = 0$ Or $6x^2 - 13x + 6 = 0$

Which gives $x = \frac{3}{2}$ or $\frac{2}{3}$.

Since $C_2 > C_1$, $x = \frac{2}{3}$ is not possible.

45. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(a) $3C_1 = 5C_2$ (b) $3C_1 + 5C_2 = 0$ (c) $9C_1 = 4C_2$ (d) $5C_1 = 3C_2$

(a)

Common potential, $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

V = 0 only if $C_1V_1 = C_2V_2$

$120C_1 = 200C_2$

$6C_1 = 10C_2$

$3C_1 = 5C_2$.

46. The energy stored in a parallel plate condenser of plate separation 'd' and plate area of cross-section A such that the uniform electric field between the plates is E.

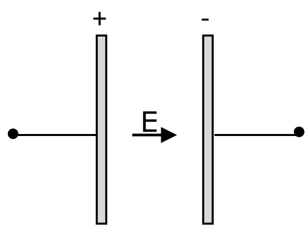
(a) $\in_0 E^2 Ad$ (b) $\frac{1}{2} \in_0 E^2 Ad$ (c) $\frac{1}{2}\frac{\in_0 E^2}{Ad}$ (d) $\frac{\in_0 E^2}{Ad}$

(b)

Energy density $u = \frac{1}{2} \in_0 E^2$

Volume of capacitor $V' = Ad$

Energy stored in capacitor $U = uV' = \frac{1}{2} \in_0 A^2 d$



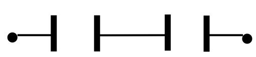
47. In how many ways one can arrange three identical capacitors taking either one or two or three capacitors together to obtain distinct effective capacitance is

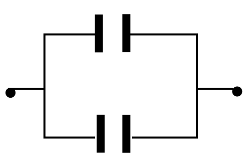
(a) 4 (b) 5 (c) 6 (d) 7

(d)

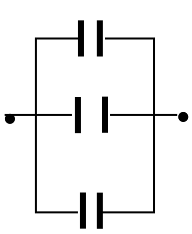
The various possible arrangements are

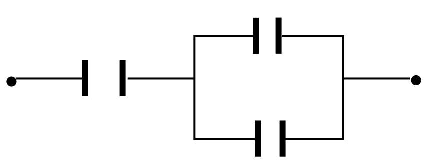
a) 

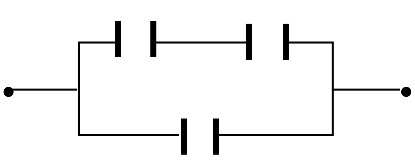
b) 

c) 

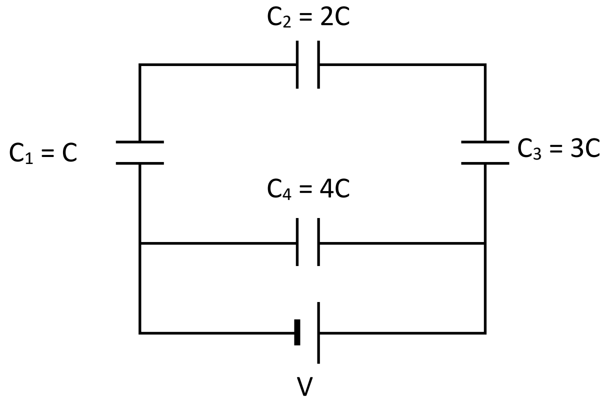
d) 

e) 

f) 

g) 

48. A network of four capacitors of capacity equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected with a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is



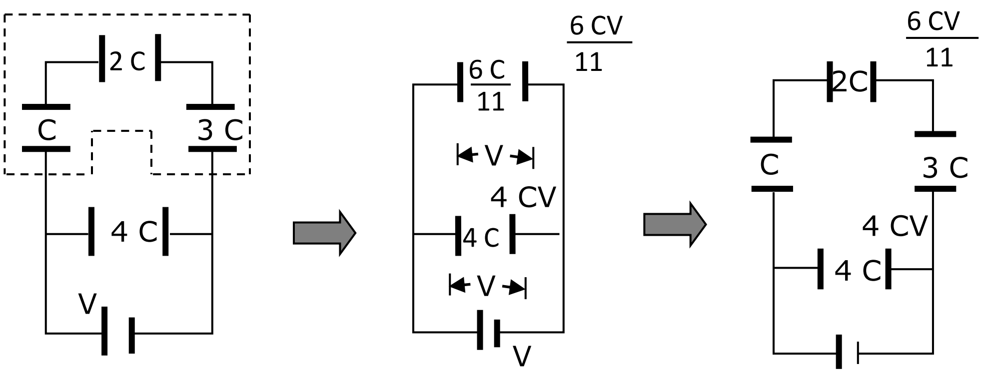
(a) $\frac{22}{3}$ (b) $\frac{3}{22}$

(c) $\frac{7}{4}$ (d) $\frac{4}{7}$

(b)

Series, $\frac{1}{C'} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$

$C' = \frac{6C}{11}$



Ratio of charges on C_2 and C_4 is $\frac{(6CV/11)}{4CV} = \frac{3}{22}$.

49. A parallel plate capacitor is connected to a cell. A metal plate of negligible thickness is inserted parallel to plates.

(A) The capacitances remains same

(B) The cell will supply more charge

(C) Equal and opposite charges will appear on two faces of metal plate.

(D) The potential difference between the plates will increase

(a) A, B (b) A, C (c) B, C (d) C, D

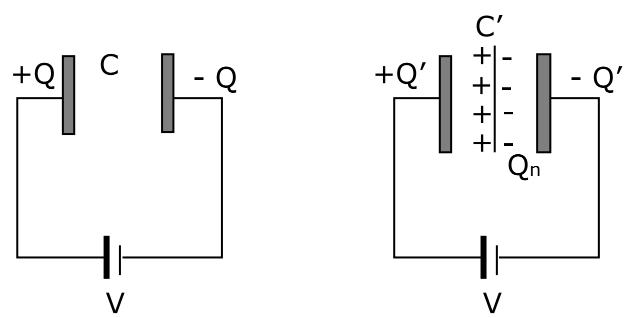
(b)

$C' = \frac{A\epsilon_0}{d - t + \frac{t}{k}}$, $t = 0, K = \infty$

$C' = \frac{A\epsilon_0}{d} = C$,(A) is correct

Q = CV, $Q' = C'V = CV$

$Q = Q'$, (B) is wrong.



Induced charge $Q_{in} = Q\left(1 - \frac{1}{K}\right) = Q\left(1 - \frac{1}{\infty}\right) = Q$ (C) is correct

Since battery is connected.

Potential difference will remain same. (D) is wrong.

50. Two identical parallel plate capacitors are connected in series to a cell of 120 V. A dielectric slab (K = 3) in placed in one of the capacitor. The p.d. across the capacitors will now be

(a) 20 V, 100 V (b) 30 V, 90 V (c) 40 v, 80 V (d) 60 V, 60 V

(b)

$V_1 = \left(\frac{3C}{C + 3C}\right) \times 120 = 90V$

$V_2 = 120 - V_1 = 30V$.