I PUC – CHAPTER-07
SYSTEM OF PARTICLES AND ROTATIONAL DYNAMICS

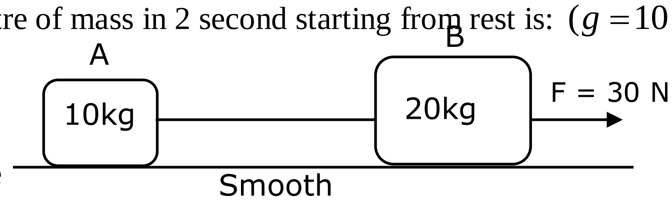
1. Two particles of equal mass have velocities $\vec{V}_1 = 8\hat{i}$ and $\vec{V}_2 = 8\hat{j}$. First particle has acceleration $\vec{a}_1 = (5\hat{i} + 5\hat{j})ms^{-2}$ while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a path of

(a) straight line (b) parabola (c) circle (d) ellipse

(a)

v_{cm} is parallel to a_{cm}

2. Two blocks A and B are connected by a massless string (shown in figure). A force of 30 N is applied on block B. The distance travelled by centre of mass in 2 second starting from rest is: $(g = 10\,ms^{-2})$



(a) 1 m (b) 2m

(c) 3 m (d) none of these

(b)

The acceleration of centre of mass is $a_{cm} = \frac{F}{m_A + m_B} = \frac{30}{10+20} = 1m/s^2$

$\therefore\ s = \frac{1}{2}a_{cm}t^2 = \frac{1}{2} \times 1 \times 2^2 = 2m$.

3. Two blocks of masses 10 kg and 30 kg are placed on X-axis. The first mass is moved on the axis by a distance of 2 cm right. By what distance should the second mass be moved to keep the position of centre of mass unchanged.

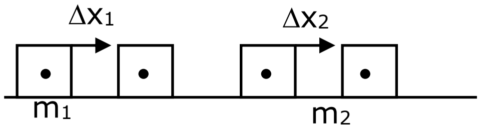
(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$

(b)

Mass of the first block, $m_1 = 10\,kg$

Mas of the second block, $m_2 = 30kg$

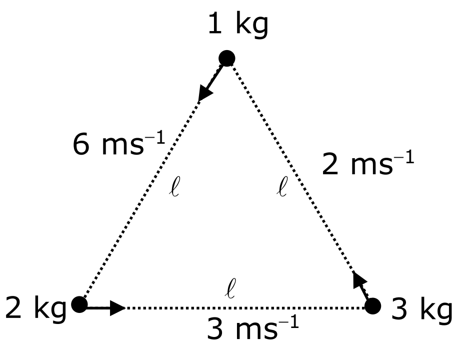
$$\Delta x_{CM} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$



$$0 = \frac{10 \times 2 + 30\Delta x}{40} \quad \therefore\ \Delta x_2 = -\frac{2}{3}$$

Therefore, the second block should be moved left through a distance of 2/3 cm to keep the position of centre of mass unchanged.

4. Three particles of masses 1kg, 2kg and 3kg are situated at the corners of an equilateral triangle move at speed $6\,ms^{-1}$, $3\,ms^{-1}$ and $2\,ms^{-1}$ respectively. Each particle maintains a direction towards the particle at the next corner symmetrically. Find velocity of CM of the system at this instant

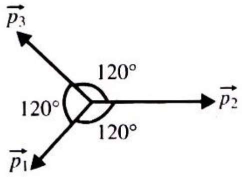


(a) $3\,ms^{-1}$ (b) $5\,ms^{-1}$ (c) $6\,ms^{-1}$ (d) zero

(d)

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3}{m_1 + m_2 + m_3}$$

$$\Rightarrow \vec{v}_{cm} = \frac{\text{Total momentum}}{\text{Total mass}}$$



Here total momentum of system is zero, because moment of each particle is same in magnitude and they are symmetrically oriented as shown.

So, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$.

5. A circular hole of radius 1 cm is cut off from a disc of radius 6 cm. The centre of hole is 3 m from the centre of the disc. The position of centre of mass of the remaining disc from the centre of disc is:

(a) $-\frac{3}{35}$ cm (b) $\frac{1}{35}$ cm (c) $\frac{3}{10}$ cm (d) none of these

(a)

For the calculation of the position of centre of mass, cut off mass is taken as negative. The mass of disc is $m_1 = \pi r_1^2\sigma = \pi 6^2\sigma = 36\pi\sigma$, where σ = surface mass density

The mass of cut portion is $m_2 = \pi(1)^2\sigma = \pi\sigma$

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

Taking origin at the centre of disc, $x_1 = 0$, $x_2 = 3cm$

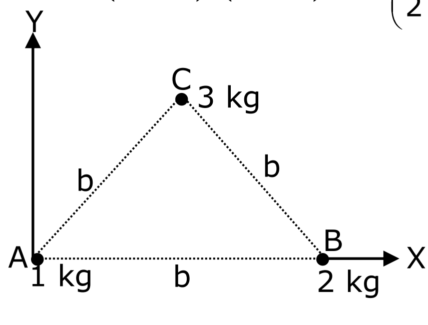
$$x_{cm} = \frac{36\pi\sigma \times 0 - \pi\sigma \times 3}{36\pi\sigma - \pi\sigma} = \frac{-3\pi\sigma}{35\pi\sigma} = -\frac{3}{35}cm$$

6. Three particles of masses 1 kg, 2 kg and 3 kg are situated at the corners of an equilateral triangle of side b. The coordinates of the centre of mass are

(a) $\left[0, \frac{7b}{12}, \frac{3\sqrt{3}b}{12}\right]$ (b) $\left[\frac{3\sqrt{3}b}{12}, \frac{7b}{12}, 0\right]$ (c) $\left[\frac{7b}{12}, \frac{3\sqrt{3}b}{12}, 0\right]$ (d) $\left[\frac{7b}{12}, 0, \frac{3\sqrt{3}b}{12},\right]$

(c)

The coordinates of points A, B and C are (0, 0, 0), (b, 0, 0) and $\left(\frac{b}{2}, \frac{b\sqrt{3}}{2}, 0\right)$ respectively.



Now as the triangle in XY plane, i.e., Z coordinate of all the masses is zero, so $Z_{CM} = 0$

Now, $X_{CM} = \frac{1 \times 0 + 2 \times b + 3(b/2)}{1 + 2 + 3} = \frac{7b}{12}$

$$Y_{CM} = \frac{1 \times 0 + 2 \times 0 + 3\sqrt{3}(b/2)}{1 + 2 + 3} = \frac{3\sqrt{3}b}{12}$$

So the coordinates of centre of mass are $\left[\frac{7b}{12}, \frac{3\sqrt{3}b}{12}, 0\right]$.

7. Two particles of mass 1 kg and 3 kg have position vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $-2\hat{i}+3\hat{j}-4\hat{k}$ respectively. The centre of mass has a position vector

(a) $\hat{i}+3\hat{j}-2\hat{k}$ (b) $-\hat{i}-3\hat{j}-2\hat{k}$ (c) $-\hat{i}+3\hat{j}+2\hat{k}$ (d) $-\hat{i}+3\hat{j}-2\hat{k}$

(d)

Here, $m_1 = 1$ kg, $m_2 = 3$ kg, $\vec{r}_1 = 2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{r}_2 = -2\hat{i}+3\hat{j}-4\hat{k}$

The position vector of the centre of mass is $\vec{r}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$

$$= \frac{(1)(2\hat{i}+3\hat{j}+4\hat{k})+(3)(-2\hat{i}+3\hat{j}-4\hat{k})}{1+3} = \frac{2\hat{i}+3\hat{j}+4\hat{k}-6\hat{i}+9\hat{j}-12\hat{k}}{4}$$

$$= \frac{-4\hat{i}+12\hat{j}-8\hat{k}}{4} = -\hat{i}+3\hat{j}-2\hat{k}.$$

8. If two particles of masses 3kg and 6 kg which are at rest are separated by a distance of 15 m. The two particles are moving towards each other under a mutual force of attraction. Then the ratio of distances travelled by the particles before collision is

(a) 2: 1 (b) 1: 2 (c) 1: 3 (d) 3: 1

(a)

In the absence of external force, centre of mass of a body remains same.

Using law of moments of mass, $m_1r_1 = m_2r_2$

$3r_1 = 6r_2$

$\frac{r_1}{r_2} = \frac{2}{1}$

9. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time?

(a) 11.2 m (b) 13.8 m (c) 14.3 m (d) 15.4 m

(c)

Centre of mass will not move in horizontal direction.

Let 'x' be the displacement of boat.

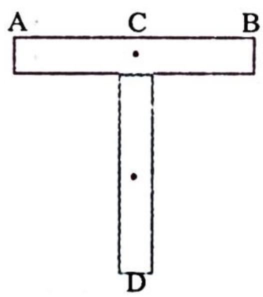
$m_1r_1 = m_2r_2$

$80(8-x) = 200x$

$\Rightarrow 640 - 80x = 200x \Rightarrow x = 2.3m$

Now required distance from the shore= $20-(8-x) = 20-(8-2.3) = 14.3m$.

10. Two identical thin uniform rods of length L each are joined to form T shape as shown in the figure. The distance of centre of mass from D is



(a) 0

(b) L/4

(c) 3L/4

(d) L

(c)

$$X_{cm} = \frac{m\left(\frac{L}{2}\right)+m(L)}{2m} = \frac{3}{4}L.$$

11. A solid sphere of mass 500g and radius 10 cm rolls down on an inclined plane without slipping. The height of the centre of mass of the sphere from the ground is 0.8 m. The translational speed of the centre of mass of the sphere on reaching the bottom of inclined will be ($g = 10\ ms^{-2}$)

(a) $\sqrt{5}$ m/s (b) $\sqrt{6}$ m/s (c) $\sqrt{10}$ m/s (d) 20 m/s

(c)

Height of the incline plane $h = h_{cm} - R = 0.8 - 0.1 = 0.7$ m

$$mg(0.7) = \frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\omega^2$$

$$mg(0.7) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2mR^2}{5}\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}mv^2$$

$$v = \sqrt{10}\, m/s$$

12. A horizontal rod rotates about a vertical axis through one end. A ring, which can slide along the rod without friction, is initially close to the axis and then slide to the other end of the rod. In this process, which of the following quantities will be conserved?

[L = angular momentum, K_T = total kinetic energy, K_R = rotational kinetic energy]

(a) L only (b) L and K_T only (c) L and K_R only d) K_T only

(b)

As no external torque acts on the system, its angular momentum is conserved. Since there is no loss of energy due to friction, the total energy is conserved. However, the ring acquires some translational kinetic energy as it slides outwards, and hence rotational kinetic energy is not conserved.

13. The ratio of radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis perpendicular to plane of disc or ring is

(a) 1 : 2 (b) $\sqrt{5} : \sqrt{6}$ (c) 2 : 3 (d) $\sqrt{3} : 2$

(d)

Radius of gyration $K = \sqrt{\frac{I}{m}}$

$$K_{disc} = \sqrt{\frac{\frac{1}{2}mR^2 + mR^2}{m}} = \sqrt{\frac{3}{2}}R$$ (Using parallel axes theorem)

$$K_{ring} = \sqrt{\frac{mR^2 + mR^2}{m}} = \sqrt{2}R$$

$$\frac{K_{disc}}{K_{ring}} = \frac{\sqrt{\frac{3}{2}}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$$

14. A body is rotating with angular velocity $\vec{\omega} = \left(3\hat{i} - 4\hat{j} + \hat{k}\right)$. The linear velocity of a point having position vector $\vec{r} = \left(5\hat{i} - 6\hat{j} + 6\hat{k}\right)$ is

(a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (b) $18\hat{i} + 3\hat{j} - 2\hat{k}$ (c) $-18\hat{i} - 13\hat{j} + 2\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

(c)

Here, $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$, $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

As $$\vec{\upsilon} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix} = \hat{i}\left(-24 - (-6)\right) - \hat{j}\left(18 - 5\right) + \hat{k}\left(-18 - (-20)\right) = -18\hat{i} - 13\hat{j} + 2\hat{k}$$

15. A circular platform is mounted on a vertical frictionless axle. Its radius is 2 m and its moment of inertia is 200 kg m^2. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed v_0 = 1.0 ms^{-1} relative to the ground. The angular velocity of the platform is

(a) 1.2 rad s^{-1} (b) 0.4 rad s^{-1} (c) 0.7 rad s^{-1} (d) 2.0 rad s^{-1}

(c)

As the system is initially at rest, initial angular momentum, $L_i = 0$.

According to the law of conservation of angular momentum, final angular momentum, $L_f = 0$.

$\therefore$ Angular momentum of man = Angular momentum of platform in opposite direction.

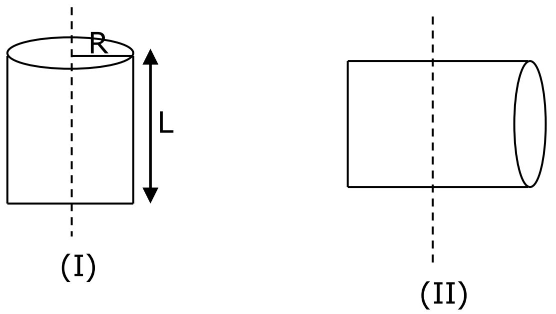
i.e., $mv_0r = I\omega$

or $\omega = \frac{mv_0r}{I} = \frac{70(1.0)(2)}{200}$ = 0.7 rad s^{-1}.

16. A uniform solid cylinder has a radius R and length L. If the moment of inertia of this cylinder about an axis passing through its centre and normal to its circular face is equal to the moment of inertia of the same cylinder about an axis passing through its centre and perpendicular to its length, then

(a) L = R (b) $L = \sqrt{3}$ R (c) $L = \frac{R}{\sqrt{3}}$ (d) $L = \sqrt{\frac{3}{2}}R$

(b)

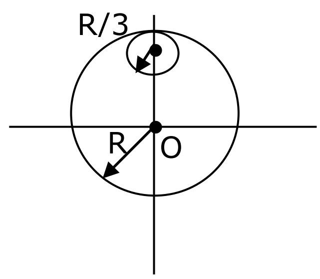


(I) Moment of inertia of a cylinder about an axis passing through its centre and normal to its circular face = $\frac{MR^2}{2}$

(II) Moment of inertia of the same cylinder about an axis passing through its centre and perpendicular to its length = $M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$

According to question $\frac{ML^2}{12} + \frac{MR^2}{4} = \frac{MR^2}{2}$ or $L = \sqrt{3}R$

17. From a circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$
(b) $\frac{40}{9}MR^2$
(c) $40MR^2$
(d) $\frac{37}{9}MR^2$

(a)

Mass per unit area of disc = $\frac{9M}{\pi R^2}$

Mass of removed portion of disc = $\frac{9M}{\pi R^2}$ x $\pi\left(\frac{R}{3}\right)^2 = M$

Moment of inertia of removed portion about an axis passing through centre of disc and perpendicular to the plane of disc, using theorem of parallel axes is $I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2}MR^2$

When portion of the disc would not have been removed, then the moment of inertia of complete disc about the given axis is $I_2 = \frac{9}{2}MR^2$

So, moment of inertia of the disc with removed portion, about the given axis is $I = I_2 - I_1 = \frac{9}{2}MR^2 - \frac{1}{2}MR^2 = 4MR^2$

18. A thin rod of length L and mass M is bent at its midpoint into two halves so that the angle between them is 90°. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is

(a) $\frac{ML^2}{24}$ (b) $\frac{ML^2}{12}$ (c) $\frac{ML^2}{6}$ (d) $\frac{\sqrt{2}ML^2}{24}$

(b)

Since rod is bent at the middle, so each part of it will have length $\left(\frac{L}{2}\right)$ and mass $\left(\frac{M}{2}\right)$

Moment of inertia through of each part through its one end = $\frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{ML^2}{24}$

Hence, moment of inertia of bend rod about an axis passing through O and perpendicular to the plane is I= $2\left(\frac{ML^2}{24}\right) = \frac{1}{12}ML^2$

19. A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity 4m s^{-1}. It collides with a horizontal spring of force constant 200 N m^{-1}. The maximum compression produced in the spring will be

(a) 0.5 m (b) 0.6 m (c) 0.7 m (d) 0.2 m

(b)

Here, m = 3kg, v = 4 ms^{-1}, k = 200 N m^{-1}, x = ?

The compression will be maximum, when

Gain in potential energy of spring = Loss in total kinetic energy of rolling cylinder

$$\frac{1}{2}kx^2 = \frac{1}{2}m\upsilon^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m\upsilon^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 \quad (\because \text{For solid cylinder } I = \frac{1}{2}mr^2)$$

$$= \frac{1}{2}m\upsilon^2 + \frac{1}{4}m\upsilon^2 \quad (\because v = r\omega)$$

$$\frac{1}{2}kx^2 = \frac{3}{4}m\upsilon^2$$

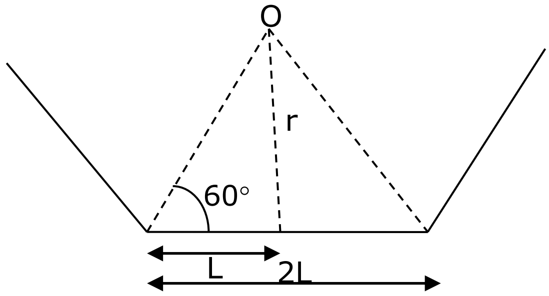
$$x = \sqrt{\frac{3}{2}\frac{m}{k}} x\upsilon = \sqrt{\frac{3}{2} x \frac{3}{200}} x4 = \frac{3x4}{20}$$

x = 0.6 m

20. A uniform thin bar of mass 6m and length 12L is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is

(a) $20mL^2$ (b) $6mL^2$ (c) $\frac{12}{5}mL^2$ (d) $30mL^2$

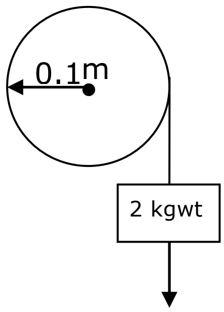
(a)
Length of each side of hexagon = 2 L
Mass of each side = m
Let O be centre of mass of hexagon. Therefore, perpendicular distance of O from each side r = L tan 60° = $L\sqrt{3}$



The desired moment of inertia of hexagon about O is $I = 6[I_{\text{one side}}]$

$$I = 6\left[\frac{m(2L)^2}{12} + mr^2\right] = 6\left[\frac{mL^2}{3} + m\left(L\sqrt{3}\right)^2\right] = 6\left[\frac{mL^2}{3} + 3mL^2\right] = 20mL^2$$

21. The moment of inertia of a solid flywheel about its axis is 0.1 kg m^2. A tangential force of 2 kg wt. is applied round the circumference of the flywheel with the help of a string and mass arrangement as shown in the figure. If the radius of the wheel is 0.1 m, find the acceleration of the mass (g = 9.8 m/s^2)



(a) 163.3 rad s^{-2}

(b) 16.3 rad s^{-2}

(c) 81.66 rad s^{-2}

(d) 8.16 rad s^{-2}

(b)
Suppose a be the linear acceleration of the mass and T the tension in the string.
Hence, Mg – T = Ma ...(i)
Let α be the angular acceleration of the flywheel. The couple applied to the flywheel is $I\alpha = TR$ or $T = \frac{I\alpha}{R}$ ...(ii)
Now we know that $a = R\alpha$ ...(iii)
Putting (ii) and (iii) in eq. (i), we get $Mg - \frac{I\alpha}{R} = MR\alpha$

$$\therefore \alpha = \frac{MgR}{I + MR^2} = \frac{2 \times 9.8 \times 0.1}{0.1 + 2 \times (0.1)^2} = 16.33 \text{ rad s}^{-1}.$$

22. A thin circular disc of mass M and radius R is rotating in a horizontal plane bout an axis passing through its centre and perpendicular to its plane with an angular velocity ω. If another disc of same dimensions but of mass M/4 is placed gently on the first disc co-axially, then the new angular velocity of the system is

(a) $\frac{5}{4}\omega$ (b) $\frac{2}{3}\omega$ (c) $\frac{4}{5}\omega$ (d) $\frac{3}{2}\omega$

(c)
According to conservation of angular momentum
$I_1\omega_1 = I_2\omega_2$ ($\because$ here $\omega_1 = \omega$)

$$\frac{1}{2}MR^2\omega = \left[\frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2\right]\omega_2$$

$\therefore\ \omega_2 = \frac{4}{5}\omega$

23. A flywheel rotates with a uniform angular acceleration. Its angular velocity increases from 20π rad s^{-1} to 40π rad s^{-1} in 10 s. how many rotations did it make in this period?

(a) 80 (b) 100 (c) 120 (d) 150

(d)

Here, $\omega_1 = 20\pi$ rad s^{-1}, $\omega_2 = 40\pi$ rad s^{-1}, t = 10 s

As $\omega_2 = \omega_2 + \alpha t$

$\therefore\ 40\pi = 20\pi + \alpha \times 10$

$\alpha = 2\pi$ rad s^{-2}

From $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

$(40\pi)^2 - (20\pi)^2 = 2 \times 2\pi\theta$

$$\theta = \frac{1200\pi^2}{4\pi} = 300\pi$$

Number of rotations completed = $\frac{\theta}{2\pi} = \frac{300\pi}{2\pi} = 150$

24. An electric motor drill, rated 350 W has an efficiency of 35%. The torque produced, if it working at 3000 rpm is

(a) 0.21 N m (b) 0.6 N m (c) 0.39 N m (d) 0.9 N m

(c)

Angular velocity of the motor drill, $\omega = \frac{2\pi f}{60} = \frac{2\pi \times 3000}{60} = 100\pi$

Let τ be the torque produced by the motor.

Power produced = $\tau\omega = \tau \times 100\pi$

Now, $\tau \times 100\pi$ = 35% of 350 W

Or $\tau \times 100\pi = \frac{35}{100} \times 350$

$$\tau = \frac{35 \times 350}{100 \times 100\pi} = 0.39 \text{ N m}$$

25. A rope is wound round a hollow cylinder of mass 3 kg and radius 40 cm. If the rope is pulled with a force of 30 N, angular acceleration of the cylinder will be

(a) 10 rad s^{-2} (b) 15 rad s^{-2} (c) 20 rad s^{-2} (d) 25 rad s^{-2}

(d)

Moment of inertia of the hollow cylinder about its axis is

$I = MR^2 = 3 \times (0.40)^2 = 0.48$ kg m^2

Torque applied on the cylinder, τ = force x moment arm = 30 x 0.40 = 12 N m

Angular acceleration of the cylinder, $\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25$ rad s^{-2}.

26. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 6000 joule, an angular acceleration of 25 rad s^{-2} must be applied about that axis for duration of

(a) 4 s (b) 2 s (c) 8 s (d) 10 s

(a)

Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\alpha t)^2 = \frac{1}{2}I\alpha^2 t^2$

Substituting the given values, we get $6000 = \frac{1}{2} \times 1.2 \times (25)^2 \times t^2$

Or $t^2 = 16$ or $t = 4$ s

27. Three point masses m_1, m_2, m_3 ate located at the vertices of an equilateral triangle of side a. The moment of inertia of system about an axis along the altitude of the triangle passing through m_1 is

(a) $(m_2 + m_3)\frac{a^2}{4}$ (b) $(m_1 + m_2 + m_3)a^2$

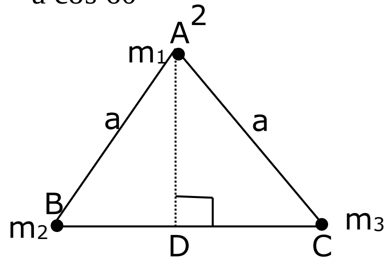
(c) $(m_1 + m_2)\frac{a^2}{2}$ (d) $(m_2 + m_3)a^2$

(a)

Perpendicular distance of m_2 and m_3 from altitude of triangle = a cos 60° = $\frac{a}{2}$

Moment of inertia of the system about the given axis is

$I = m_1 \times 0 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2 = (m_2 + m_3)\frac{a^2}{4}$



28. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

(a) $\frac{2R}{\sqrt{15}}$ (b) $R\sqrt{\frac{2}{15}}$ (c) $\frac{4R}{\sqrt{15}}$ (d) $\frac{R}{4}$

(a)

Moment of inertia of solid sphere of mass M and radius R about an axis passing through the centre of mass is $I = \frac{2}{5}MR^2$.

Let the radius of the disc be r. Moment of inertia of circular disc of radius r and mass M about an axis passing through the centre of mass and perpendicular to its plane = $\frac{1}{2}Mr^2$.

Using theorem of parallel axes, moment of inertia of disc about its edge is

$I' = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$

Given I = I'

$\therefore \frac{2}{5}MR^2 = \frac{3}{2}Mr^2$

$r^2 = \frac{4}{15}R^2$ or $r = \frac{2R}{\sqrt{15}}$

29. Two rings of radius R and nR made up of same material have the ratio of moment of inertia about an axis passing through centre as 1 : 8. The value of n is

(a) 2 (b) $2\sqrt{2}$ (c) 4 (d) $\frac{1}{2}$

(a)

The moment of inertia of circular ring whose axis of rotation is passing through its centre is, $I = mR^2$

$\therefore I_1 = m_1R^2$ and $I_2 = m_2(nR)^2$

Since, both have same density $\therefore \frac{m_2}{2\pi(nR) \times A} = \frac{m_1}{2\pi R \times A}$

Where A is cross – section area of ring $\therefore m_2 = nm_1$

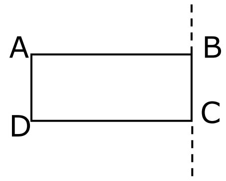
$\because \frac{I_1}{I_2} = \frac{m_1R^2}{m_2(nR)^2} = \frac{m_1R^2}{m_1n(nR)^2} = \frac{1}{n^3}$

$\because \frac{I_1}{I_2} = \frac{1}{8}$ given $\therefore \frac{1}{8} = \frac{1}{n^3}$ or n = 2.

30. A wire of length ℓ and mass m is bent in the form of a rectangle ABCD with $\frac{AB}{BC} = 2$. The moment of inertia of this wire frame about the side BC is

(a) $\frac{11}{252}m\ell^2$ (b) $\frac{8}{203}m\ell^2$ (c) $\frac{5}{136}m\ell^2$ (d) $\frac{7}{162}m\ell^2$

(d)



$\frac{AB}{BC} = 2 \therefore AB = DC = \frac{\ell}{3}$

and $BC = AD = \frac{\ell}{6}$

Similarly, $m_{AB} = m_{DC} = \frac{m}{3}$ and $m_{BC} = m_{AD} = \frac{m}{6}$

$\therefore$ Moment of inertia of the wire frame about the given axis is

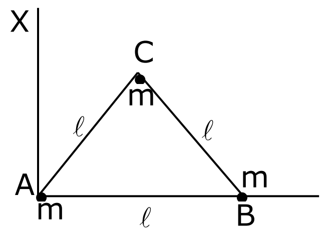
$I = I_{AB} + I_{AD} + I_{DC} + I_{BC} = \frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2 + \left(\frac{m}{6}\right)\left(\frac{\ell}{3}\right)^2 + \frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2 + 0$

$= \frac{m\ell^2}{81} + \frac{m\ell^2}{54} + \frac{m\ell^2}{81} = \frac{7}{162}m\ell^2$

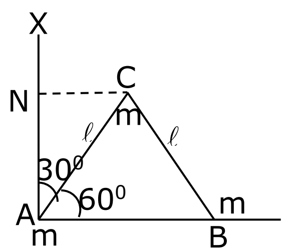
31. Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side ℓ cm as shown in figure. The moment of inertia of the system about a line AX Perpendicular to AB and in the plane of ABC, in gram cm^2 will be

(a) $\frac{5}{4}ml^2$ (b) $\frac{3}{2}ml^2$

(c) $\frac{3}{4}ml^2$ (d) $2ml^2$



(a)



From figure, $\sin 30^0 = \frac{CN}{CA} = \frac{CN}{\ell}$

$CN = \ell \sin 30^0 = \frac{\ell}{2}$

M.I of the system about the desired axis is

$I = m_A(0)^2 + m_B(\ell)^2 + m_c(CN)^2 = 0 + m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{5}{4}m\ell^2$

32. A mass M is moving with a constant velocity parallel to the x–axis. Its angular momentum w.r.t the origin

(a) is zero (b) remains constant

(c) goes on increasing (d) goes on decreasing

(b)

33. A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of α for which angular momentum about origin is conserved is

(a) 1 (b) – 1 (c) 2 (d) zero

(b)

Angular momentum, L = constant, when $\vec{\tau} = \vec{r} \times \vec{F} = 0$

i.e. $(2\hat{i} - 6\hat{j} - 12\hat{k}) \times (\alpha\hat{i} + 3\hat{j} + 6\hat{k}) = 0$ or $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix} = 0$, which gives $\alpha = -1$

34. A flywheel of mass 50 kg and radius of gyration about its axis of rotation of 0.5 m is acted upon by a constant torque of 12.5 Nm. Its angular velocity at t = 5 s is

(a) 2.5 rad/s (b) 5 rad/s (c) 7.5 rad/s (d) 10 rad/s

(b)

$I = Mk^2 = 50 \times (0.5)^2$ kgm^2

$\tau = 12.5$ Nm

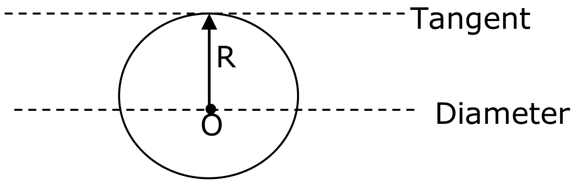
$\alpha = \frac{\tau}{I} = \frac{12.5}{50 \times \left(\frac{1}{2}\right)^2} = 1$ rad/s^2

$\omega = \omega_0 + \alpha t = 0 + 1 \times 5 = 5$ rad/s

35. The moment of inertia of a disc, of mass M and radius R, about an axis which is a tangent and parallel to its diameter is

(a) $\frac{1}{2}MR^2$ (b) $\frac{3}{4}MR^2$ (c) $\frac{1}{4}MR^2$ (d) $\frac{5}{4}MR^2$

(d)



Using the theorem of parallel axes, $I_{tangent} = I_{diameter} + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$

36. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotation will be

(a) $\frac{K^2 + R^2}{R^2}$ (b) $\frac{K^2}{R^2}$ (c) $\frac{K^2}{K^2 + R^2}$ (d) $\frac{R^2}{K^2 + R^2}$

(c)

Total kinetic energy = K.E. of translation + K.E of rotation

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}MK^2\frac{v^2}{R^2} = \frac{1}{2}Mv^2\left[1 + \frac{K^2}{R^2}\right] (\because I = MK^2 \text{ and } v = \omega R)$$

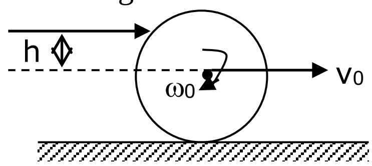
$$\frac{\text{KE of roatation}}{\text{Total KE}} = \frac{\frac{1}{2}MK^2\frac{v^2}{R^2}}{\frac{1}{2}Mv^2\left[1 + \frac{K^2}{R^2}\right]} = \frac{K^2}{K^2 + R^2}$$

37. A billiard ball is hit by a cue at a height h above the centre. It acquires a linear velocity v_0. Mass of the ball is m and radius is r. The angular velocity ω_0 acquired by the ball is

(a) $\frac{2v_0h}{5r^2}$ (b) $\frac{5v_0h}{2r^2}$ (c) $\frac{2v_0r^2}{5h}$ (d) $\frac{5v_0r^2}{2h}$

(b)

When the ball is hit by a cue, the linear impulse imparted to the ball = change in momentum = mv_0



As is clear from figure,

Angular momentum = moment of momentum

$I\omega_0 = (mv_0)\, h$

$\left(\frac{2}{5}mr^2\right)\omega_0 = (mv_0)h$

$\therefore\ \omega_0 = \frac{5v_0 h}{2r^2}$

38. Circular disc of mass 2 kg and radius 1 m is rotating about an axis perpendicular to its plane and passing through its centre of mass with a rotational kinetic energy of 8J. The angular momentum in (Js) is

(a) 8 (b) 4 (c) 2 (d) 1

(b)

Here, Mass of the disc M = 2kg, Radius of the disc R = 1m

Moment of inertia of the disc about an axis perpendicular to its plane and passing through its centre of mass is $I = \frac{1}{2}MR^2 = \frac{1}{2} x (2)(1)^2 = 1 kg\ m^2$

Kinetic energy of rotation, $K_R = \frac{L^2}{2I}$ where L is the angular momentum

or L = $\sqrt{2K_R I} = \sqrt{2 x 8 x 1} = 4\ Js$

39. Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

(a) 1 : 2 (b) $\sqrt{2} : 1$ (c) $1 : \sqrt{2}$ (d) 2 :1

(c)

L = $\sqrt{2K_R I}$

Since K is same, $\frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$ (Uisng (i))

$\frac{L_1}{L_2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

40. Angular momentum of the particle rotating with a central force is constant due to

(a) Constant torque (b) constant force

(c) Constant linear momentum (d) zero torque

(d)

Torque due to central force is zero.

As $\tau = \frac{dL}{dt} = 0$ $\therefore$ L = constant

41. The diameter of a flywheel is 1m. It has a mass of 20kg. It is rotating about its axis with seed of 120 rotations in one minute. Its angular momentum in kg m^2s^{-1} is

(a) 13.4 (b) 31.4 (c) 41.4 (d) 43.4

(b)

Here, Diameter of flywheel, D = 1 m

Radius of the flywheel, $R = \frac{D}{2} = \frac{1}{2} m$

Mass of the flywheel, M = 20kg

Angular speed of rotation, $\omega = 120\, rpm = \frac{2\pi x 120}{60}$ rad s^{-1} = 4π rad s^{-1}

Moment of inertia of the flywheel about its axis, $I = \frac{1}{2}MR^2 = \frac{1}{2}(20)\left(\frac{1}{2}\right)^2$ = 2.5 kg m^2

Angular momentum, L = $I\omega$ = (2.5) (4π) = 10π = 31.4 kg m^2 s^{-1}

42. A carpet of mass M, made of an extensible material is rolled along its length in the form of a cylinder of radius R and kept on a rough floor. If the carpet is unrolled, without sliding to a radius R/2, the decrease in potential energy is

(a) $\frac{1}{2}MgR$ (b) $\frac{7}{8}MgR$ (c) $\frac{5}{8}MgR$ (d) $\frac{3}{4}MgR$

(b)

The centre of mass of the whole carpet is originally at a height R above the floor. When the carpet unrolls itself and has radius R/2, the centre of mass is at a height R/2.

The mass left over unrolled = $\frac{M\pi(R/2)^2}{\pi R^2} = \frac{M}{4}$

Decrease in potential energy = $MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) = \frac{7}{8}MgR$

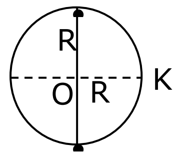
43. Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. The moment of inertia of this system about an axis passing through one of the particle and perpendicular to plane of disc is

(a) $16mR^2$ (b) $(3M + 16m)\frac{R^2}{2}$ (c) $(3m+12M)\frac{R^2}{2}$ (d) zero

(b)

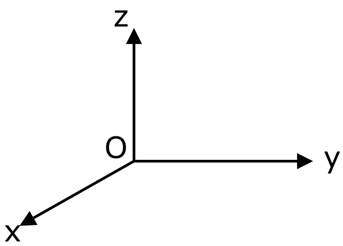
According to the theorem of parallel axes, moment of inertia of disc about an axis passing through K and perpendicular to plane of disc is $= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

Total moment of inertia of the system = $\frac{3}{2}MR^2 + m(2R)^2 + m(\sqrt{2}R)^2 + m(\sqrt{2}R)^2$



$I = (3M + 16m)\frac{R^2}{2}$

44. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, - 1) is



(a) $-F(\hat{i}+\hat{j})$ (b) $F(\hat{i}+\hat{j})$

(c) $-F(\hat{i}-\hat{j})$ (d) $F(\hat{i}-\hat{j})$

(b)

Here, $\vec{F} = -F\hat{k}$; $\vec{r} = (\hat{i}-\hat{j})$

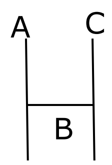
$\therefore\ \vec{\tau} = \vec{r} \times \vec{F} = (\hat{i}-\hat{j}) \times (-F\hat{k}) = -F(\hat{i}\times\hat{k} - \hat{j}\times\hat{k}) = -F(-\hat{j}-\hat{i}) = F(\hat{i}+\hat{j})$

45. Three identical thin rods each of length l and mass M are joined together to form a letter H. What is the moment of inertia of the system about one of the sides of H?

(a) $\frac{Ml^2}{6}$ (b) $\frac{Ml^2}{4}$ (c) $\frac{2}{3}Ml^2$ (d) $\frac{4}{3}Ml^2$

(d)

Moment of inertia of the system about rod A



$I = I_A + I_B + I_C = 0 + \left(\frac{M\ell^2}{12} + \frac{M\ell^2}{4}\right) + M\ell^2$

$= \frac{4}{3}M\ell^2$

46. The radius of gyration of a solid cylinder of mass M and radius R about its own axis is

(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{2}$ (c) $\frac{R}{\sqrt{3}}$ (d) $\frac{R}{3}$

(a)

The moment of inertia of a solid cylinder of mass M and radius R about its own axis is $I = \frac{MR^2}{2}$

As $I = MK^2$

Where K is the radius of gyration

$K^2 = \frac{R^2}{2}$ or $K = \frac{R}{\sqrt{2}}$

47. If the radius of a solid sphere is 35 cm, calculate the radius of gyration when the axis is along a tangent:

(a) $7\sqrt{10}$ cm (b) $7\sqrt{35}$ cm (c) $\frac{7}{5}$ cm (d) $\frac{2}{5}$ cm

(b)

MI about tangent, $I = I_{CG} + MR^2 = \frac{2}{5}MR^2 + MR^2$

or $MK^2 = \frac{7}{5}MR^2$

$\therefore K = \sqrt{\frac{7}{5}}R = \sqrt{\frac{7}{5}} \times 35 \text{ cm} = 7\sqrt{35}$ cm

48. The moment of inertia of a uniform thin rod of length L and mass M about an axis passing through a point at a distance of L/3 from one of its ends and perpendicular to the rod is

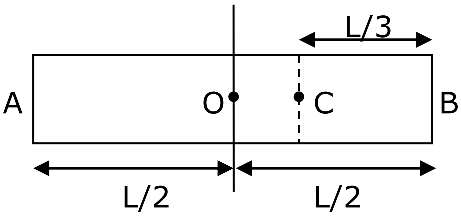
(a) $\frac{7ML^2}{48}$ (b) $\frac{ML^2}{1}$ (c) $\frac{ML^2}{9}$ (d) $\frac{ML^2}{3}$

(c)

The distance $OC = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$

Applying the theorem of parallel axes,

$I_C = I_O + M(OC)^2 = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$



49. A uniform disc of radius R lies in XY-plane with its centre at origin. Its moment of inertia about the axis x = 2R and y = 0 is equal to the moment of inertia about the axis y = d and z = 0, where d is equal to

(a) $\frac{4}{3}$ R (b) $\frac{\sqrt{17}}{2}$ R (c) $\sqrt{13}$ R (d) $\frac{\sqrt{15}}{2}$ R

(b)

An axis passing through x = 2R, y = 0 is in ⊗ direction as shown in the figure.

Moment of inertia about this axis will be,

$I_1 = \frac{1}{2}mR^2 + m(2R)^2 = \frac{9}{2}mR^2$

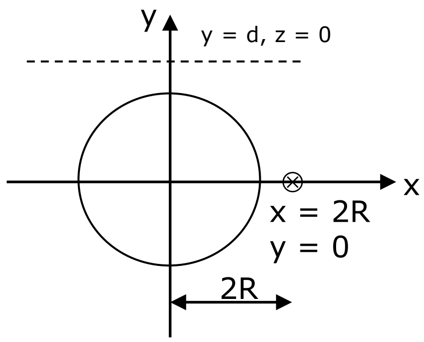
Axis passing through y = d and z = 0 is shown as dotted line in figure.

Moment of inertia about this axis will be,

$I_2 = \frac{1}{4}mR^2 + md^2$

$\therefore \frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2$

or $d = \frac{\sqrt{17}}{2}$ R



50. A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is

(a) 1 : 7 (b) 2 : 7 (c) 1 : 1 (d) 5 : 7

(b)

Total energy, $K = K_R + K_T = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2} mr^2\omega^2$

$$= \frac{1}{5} mr^2 \omega^2 + \frac{1}{2} mr^2\omega^2 = \frac{7}{10} mr^2\omega^2$$

Now, rotational kinetic energy $K_R = \frac{1}{2} I\omega^2 = \frac{1}{5} mr^2\omega^2$

$$\therefore \frac{K_R}{K} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$$