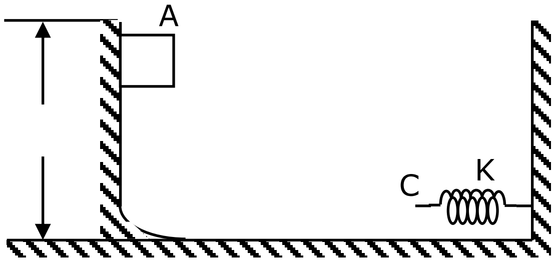
IPUC – CHAPTER 06
WORK, ENERGY AND POWER

1. A small mass 'm' is sliding down on a smooth curved inclined from a height 'h' and finally moves through a horizontal smooth surface. A light spring of force constant K is fixed with a vertical rigid stand on the horizontal surface, as shown in the figure. Then the value for the maximum compression in the spring if mass 'm' is released from rest from height 'h' and hits the spring on the horizontal surface.



(a) $\sqrt{\frac{mgh}{2K}}$ (b) $\sqrt{\frac{2mgh}{K}}$

(c) $\sqrt{\frac{3mgh}{2K}}$ (d) $\sqrt{\frac{2mgh}{3K}}$

(b)

Conservation of energy between position A and C

$(PE_A)_{block} + KE_A = (PE_C)_{spring} + KE_C$

$mgh + 0 = \frac{1}{2}Kx^2 + 0$; $mgh = \frac{1}{2}Kx^2$, $x = \sqrt{\frac{2mgh}{K}}$.

2. A vehicle of mass 15 quintal climbs up a hill 200 m high and then moves on a level road with a speed of $30\,ms^{-1}$. Its total mechanical energy while running on the top of the hill is ($g = 9.8\,ms^{-2}$)

(a) $3.615\times10^{7}\,J$ (b) $3.615\times10^{-7}\,J$ (c) $3.615\times10^{6}\,J$ (d) $3.615\times10^{-6}\,J$

(c)

m = 15 quintal = 1500 kg, $g = 9.8\,ms^{-2}$, h = 200 m

P.E. gained, U = mgh = $1500\times9.8\times200 = 2.94\times10^{6}\,J$

K.E. = $\frac{1}{2}mv^2 = \frac{1}{2}\times1500\times(30)^2 = 0.675\times10^{6}\,J$

Total mechanical energy E = K + U = $(0.675+2.94)\times10^{6} = 3.615\times10^{6}\,J$.

3. The potential energy of 1 kg particle free to move along X-axis is given by $U(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)J$. The total mechanical energy of the particle is 2 J. Then the maximum speed of the particle (in ms^{-1}) is

(a) $\frac{3}{2\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{2}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

(b)

For maximum value of U, $\frac{dU}{dx} = 0$

$\therefore \frac{4x^3}{4} - \frac{2x}{2} = 0$ or $x = 0$, $x = \pm1$

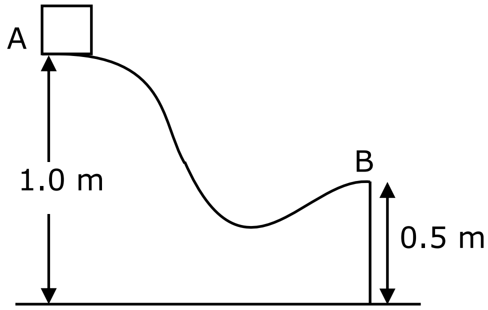
At $x = 0$, $\frac{d^2U}{dx^2} = -1$ and At $x = \pm1$, $\frac{d^2U}{dx^2} = 2$

Hence U is minimum at $x = \pm 1$ with value $U_{\min} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} J$

$K_{\max} + U_{\min} = E$ or $K_{\max} - \frac{1}{4} = 2$ or $K_{\max} = \frac{9}{4}$

$\Rightarrow \frac{1}{2}mv^2 = \frac{9}{4} \Rightarrow v_{\max} = \frac{3}{\sqrt{2}} ms^{-1}$

4. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground? (g = 9.8 ms^{-2})



(a) 1 m (b) 2m (c) 3m (d) 4m

(a)
Applying the law of conservation of mechanical energy for the point A and B,

$mgH = \frac{1}{2}mv^2 + mgh$

$g - \frac{v^2}{2} = \frac{g}{2}$ or $v^2 = g \Rightarrow v = \sqrt{g} = 3.1 ms^{-1}$

After point B the particle exhibits projectile motion with $\theta = 0^0$ and $y = -0.5m$

Horizontal distance travelled by the body $R = u\sqrt{\frac{2h}{g}} = 3.1 \times \sqrt{\frac{2 \times 0.5}{9.8}} = 1\text{m}$

5. The speed of a car changes from 0 to $5 ms^{-1}$ in the first phase and from $5 ms^{-1}$ to $10 ms^{-1}$ in the second phase and from $10 ms^{-1}$ to $15 ms^{-1}$ during the third phase. In which phase the increase in kinetic energy is more?

(a) first phase (b) second phase
(c) third phase (d) same in all the three phase

(c)

$\Delta KE = \frac{1}{2}m(v^2 - u^2)$.

$\Delta\text{KE is maximum when } (\text{v}^2 - \text{u}^2) \text{ is maximum}$

6. A particle of mass 2kg starts moving in a straight line with an initial velocity of 2 m/s at a constant acceleration of $2 ms^{-2}$. Then rate of change of kinetic energy:

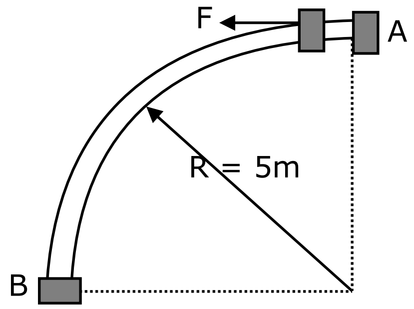
(a) Is four times the velocity at any moment
(b) Is two times the displacement at any moment.
(c) Is four times the rate of change of velocity at any moment.
(d) Is constant throughout.

(a)

$$K = \frac{1}{2}mv^2$$

$$\therefore \frac{dK}{dt} = mv\frac{dv}{dt} = \left(m\frac{dv}{dt}\right)v = (ma)v = 2\times 2\times 2 = 8J/s.$$

7. A bead of mass 0.5 kg starts from rest from 'A' to move in a vertical plane along a smooth fixed quarter ring of radius 5m, under the action of a constant horizontal force F = 5N as shown. The speed of bead as it reaches point B is (g = 10 m/s^2)



(a) 14.4 m/s

(b) 7.07 m/s

(c) 5 m/s

(d) 25 m/s

(a)

Applying the work-energy theorem, we get $\frac{1}{2}\times mv^2 - 0 = W_1 + W_2$

= Horizontal force × displacement + Vertical force × displacement

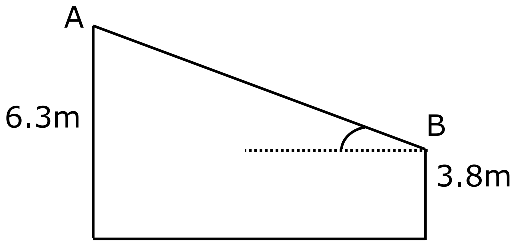
= $F\times R + mg\times R$.

½ × ½ × v^2 = 5 × 5 + 0.5 × 10 × 5

v^2 = 200

v = 14.4 m/s

8. AB is a frictionless inclined surface making an angle of 30^0 with horizontal. A is 6.3m above the ground while B is 3.8 m above the ground. A block slides down from A, initially starting from the rest. Its velocity on reaching B is (g = 9.8 m/s^2)



(a) $7ms^{-1}$ (b) $14ms^{-1}$ (c) $7.4ms^{-1}$ (d) $4.9ms^{-1}$

(a)

Gain in K.E. = loss of P.E.

½ mv^2 – 0 = = $mg(h_1 - h_2)$.

½ v^2 = 9.8 (6.3 – 3.8)

v^2 = 19.6 × 2.5 = $\frac{196\times 25}{100}$

v = $\frac{14\times 5}{10} = \frac{70}{10} = 7\text{m/s}$

9. Spring A and B are identical except that A is stiffer than B. If work expended in spring A and B are W_A and W_B, when they are stretched by same amount, while work expend in spring A and B are $W_A^{'}$ and $W_B^{'}$ when they are stretched by the same force then

(a) $W_A > W_B$ (b) $W_A < W_B$ (c) $W'_A > W'_B$ (d) $W'_A = W'_B$

(a)

Given $k_A > k_B$

$W_A = \frac{1}{2}k_A x^2$; $W_B = \frac{1}{2}k_B x^2 \Rightarrow W_A > W_B$

For same force: $F = k_A x_A \Rightarrow x_A = \frac{F}{k_A}$

$x_B = \frac{F}{K_B}$

$W_A^{'} = \frac{1}{2}k_A x_A^2 = \frac{1}{2}k_A \frac{F^2}{k_A^2} = \frac{F^2}{2k_A}$

$W_B^{'} = \frac{F^2}{2k_B}$ Þ $W_A^{'} < W_B^{'}$

10. A particle is released one by one from the top of two inclined rough surface of height h each. The angles of the inclination of the two planes are 30^o and 60^o respectively. All other factors (e.g., coefficient of friction, mass of block, etc. are same in both the cases. Let K_1 and K_2 be the kinetic energies of the particle at the bottom of the plane in the two cases. Then

(a) $K_1 = K_2$ (b) $K_1 > K_2$ (c) $K_1 < K_2$ (d) Data insufficient

(c)

Work done by friction:

$W = (\mu mg\cos\theta)s = (\mu mg\cos\theta)\frac{h}{\sin\theta} = \mu mg\,h\cot\theta$

Now, $\cot\theta_1 = \cot 30^0 = \sqrt{3}$, $\cos\theta_2 = \cot 60^0 = \frac{1}{\sqrt{3}}$

i.e., kinetic energy (KE = $mgh - W$) in first case will be less or $K_1 < K_2$.

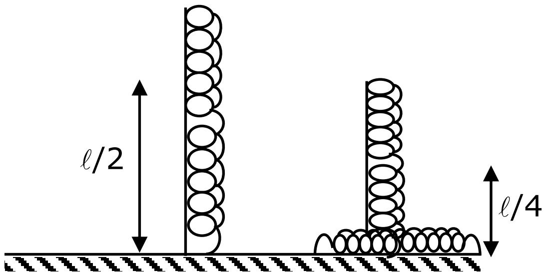
11. A man places a chain (of mass m and length ℓ) on a table slowly. Initially, the lower end of the chain just touches the table. The man brings down the chain by length $\ell/2$. Work done by the man in this process is

(a) $-mg\frac{1}{2}$ (b) $-\frac{mg\ell}{4}$ (c) $\frac{-3mg\ell}{8}$ (d) $-\frac{mg\ell}{8}$

(c)

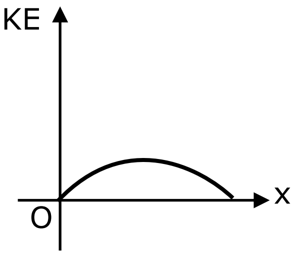
The work done by the man is negative of the magnitude of decrease in potential energy of the chain.

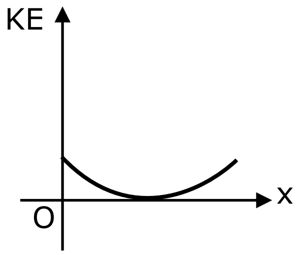
$\Delta U = mg\frac{\ell}{2} - \frac{m}{2}g\frac{\ell}{4} = 3mg\frac{\ell}{8}$

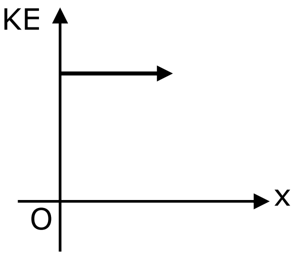


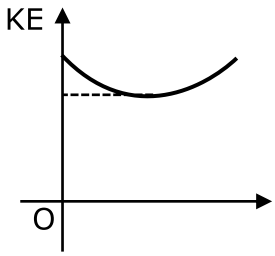
$$W = -\frac{3mg\ell}{8}.$$

12. A projectile is fired with some velocity making certain angle with the horizontal. Which of the following graphs is the best representation for the kinetic energy of a projectile (KE) versus its horizontal displacement (x)?

(a) 

(b) 

(c) 

(d) 

(d)

Velocity of a projectile at any instant of time (t) is:

$$v^2 = v_x^2 + v_y^2 = (u\cos\theta)^2 + \left(u\sin\theta - g\frac{x}{u\cos\theta}\right)^2$$

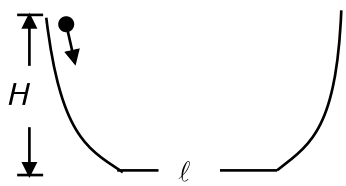
$$v^2 = (u\cos\theta)^2 + (u\sin\theta)^2 + \frac{g^2x^2}{u^2\cos^2\theta} - 2\frac{(u\sin\theta)gx}{u\cos\theta}$$

$$v^2 = u^2 + \frac{g^2x^2}{u^2\cos^2\theta} - 2gx\tan\theta$$

$\therefore$ KE = ½ mv^2 = $\frac{1}{2}mu^2 - mgx\tan\theta + \frac{mg^2x^2}{2u^2\cos^2\theta}$.

The given equation represents the equation of a parabola.

13. A particle of mass m slides along a curved- flat-curved track. The curved portions of the track are smooth. If the particle is released at the top of one of the curved portions, the particle comes to rest at flat portion of length ℓ and $\mu = \mu_{\text{kinetic}}$ after covering a distance of



(a) $\frac{\ell}{3\mu}$ (b) $\frac{H}{2\mu_{\text{kinetic}}}$

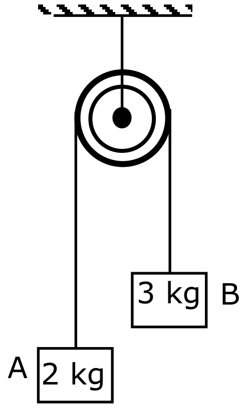
(c) $\frac{\ell}{6}$ (d) $\frac{H}{\mu_{\text{kinetic}}}$

(d)

Here PE = work done

$mgH = \mu_{\text{kinetic}} mgS$ or $S = \frac{H}{\mu_{\text{kinetic}}}$

14. If the system shown in released from rest. Then the net work done by tension in first one second ($g = 10 ms^{-2}$) is



(a) 1 J

(b) 2 J

(c) 3 J

(d) zero

(d)
For A: T and s are in same direction
For B: T and s are in opposite directions
$W_{net} = W_1 + W_2 = T \times s - T \times s = 0$

15. A machine rated as 150 W, changes the velocity of a 10 kg mass from $4 ms^{-1}$ to $10 ms^{-1}$ in 4s. The efficiency of the machine is nearly
(a) 70% (b) 30% (c) 50% (d) 40%

(a)

$$\eta = \frac{P_0}{P_1} \times 100 = \frac{\frac{1}{2} m[v^2 - u^2]}{P_1 t} \times 100 = \frac{\frac{1}{2} \times 10[10^2 - 4^2]}{150 \times 4} \times 100 = 70\%$$

16. A tank on the roof of a 20 m high building can hold $10 m^3$ of water. The tank is to be filled from a pond on the ground in 20 minutes. If the pump has an efficiency of 60%, then the input power in kW is (g = 10 m/s^2)
(a) 110 (b) 277 (c) 548 (d) 700

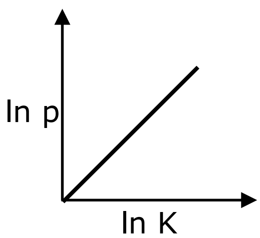
(b)

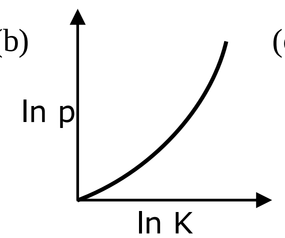
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{(mgh)/t}{P_{in}}$$

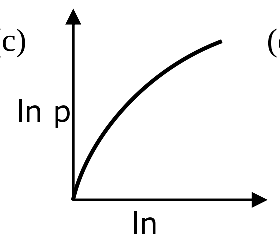
$$\frac{60}{100} p_{in} = \frac{10^3 \times 10^3 \times 10 \times 20}{20 \times 60}.$$

P_{in} = 277 kW

17. Which of the following graphs represent the graphical relation between momentum (p) and kinetic energy (K) for a body in motion?

(a)


(b)


(c)


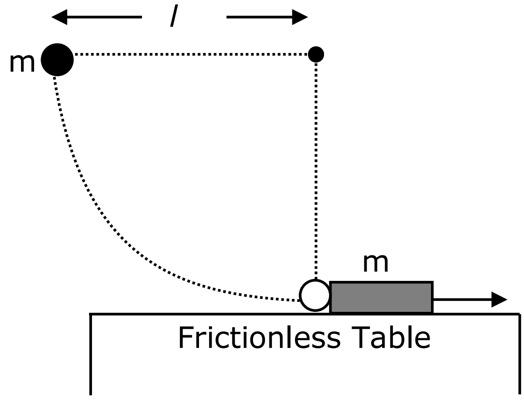
(d) None of these

(d)

$$\frac{P^2}{2m} = \text{K} \Rightarrow \ln \frac{P^2}{2m} = \ln \text{K}$$

$2\ln P - \ln(2m) = \ln K$
ln P = ½ lnK + ½ ln (2m)
y = mx + c
So, the graph between $\ln p$ and $\ln k$ is straight line with intercept.

18. The bob of a simple pendulum of length ℓ dropped from a horizontal position strikes a block of the same mass, placed on a horizontal table (frictionless) as shown in the figure, the block shall have kinetic energy



(a) zero

(b) $mg\ell$

(c) $\frac{1}{2}mg\ell$

(d) $2mg\ell$

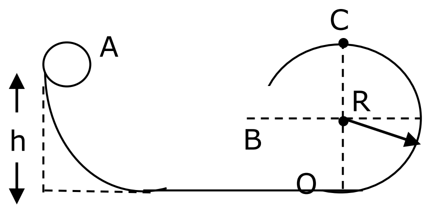
(b)

From energy conservation $mg\ell = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2g\ell}$

From momentum conservation $m\sqrt{2g\ell} = mv' \Rightarrow v' = \sqrt{2g\ell}$

K.E. = $\frac{1}{2}m \times 2g\ell = mg\ell$.

19. Ball A of mass m, after sliding from an inclined plane, strikes elastically another ball B of same mass at rest. Find the minimum height h so that ball B just completes the circular motion of the surface at C. (All surfaces are smooth).



(a) $h = \frac{5}{2}R$

(b) $h = 2R$

(c) $h = \frac{2}{5}R$

(d) $h = 3R$

(a)

$mgh = \frac{1}{2}mv^2 + 2mgR \Rightarrow gh = \frac{v^2}{2} + 2gR$

Required velocity at the top, $v = \sqrt{gR}$

$gh = \frac{gR}{2} + 2gR \Rightarrow h = \frac{5}{2}R$.

20. A block is moved from rest through a distance of 4 m along a straight line path. The mass of the block is 5 kg and the force acting on it is 20 N. If the kinetic energy acquired by the block be 40J, at what angle to the path the force is acting?

(a) 30^0 (b) 60^0 (c) 45^0 (d) none of these

(b)

F = 20 N, K.E. = 40 J

Change in K.E. = $Fs\cos\theta$

$40 = 20\times 4\cos\theta$

$\cos\theta = ½$

$\theta = 60^0$.

21. Two masses of 1 g and 9 g are moving with equal kinetics energies. The ratio of the magnitudes of their respective linear momenta is

(a) 1 : 9 (b) 9 : 1 (c) 1 : 3 (d) 3 : 1

(c)

$p = \sqrt{2mK}$

For constant K, $p \propto \sqrt{m}$ $\therefore$ $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

22. Two bodies of masses m_1 and m_2 are moving with velocities $1ms^{-1}$ and $3ms^{-1}$ respectively in opposite directions. If the bodies undergo one dimensional elastic collision, the body of mass m_1 comes to rest. Then the ratio of m_1 and m_2 is

(a) 7 : 1 (b) 1 : 7 (c) 1 : 3 (d) 3 : 1

(a)

$u_1 = 1m/s,\ u_2 = -3m/s,\ v_1 = 0$

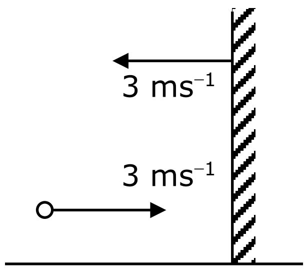
$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$ $\qquad$ $0 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)1 + \left(\frac{2m_2}{m_1 + m_2}\right)(-3)$

$m_1 - m_2 = 6m_2$

$m_1 = 7m_2$

$\frac{m_1}{m_2} = \frac{7}{1}$

23. A highly elastic ball moving at a speed of $3ms^{-1}$ approaches a wall moving towards it with a speed of $3ms^{-1}$. After the collision, the speed of the ball will be



(a) $3ms^{-1}$ (b) $6ms^{-1}$

(c) $9ms^{-1}$ (d) zero

(c)

For elastic collision, $e = 1$.

Let the speed of the ball be v towards left after collision, then w. r. t wall incident velocity = reflected velocity.

Since e = 1, $u_1 - u_2 = v_2 - v_1$

We get 3 + 3 = $v - 3$ $\Rightarrow$ $v = 9ms^{-1}$

24. Velocity of a particle is $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$ at an instant under the action of the force of $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$. The instantaneous power developed by the body in watt is

(a) 4 W (b) 6 W (c) 2 W (d) 5 W

(c)

$\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$ $\qquad$ $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{v} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ $\vec{F} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$P = \vec{F}.\vec{v}$

$P = a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 2 + (-1) \times 1 + 1 \times (-1) = 2W$

25. A bus can be stopped by applying a retarding force F when it is moving with a speed v on a level road. The distance covered by it before coming to rest is s. If the load of the bus increases by 50% because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be

(a) 1.5s (b) 2s (c) 1s (d) 2.5s

(a)

First case: $\frac{1}{2}mv^2 = Fs$

Second case: $\frac{1}{2}\left(m + \frac{m}{2}\right)v^2 = Fs'$

Dividing eqn. (ii) by eqn. (i), $\frac{s'}{s} = \frac{3}{2}$ or $s' = 1.5$ s

26. A body of mass 1 kg is thrown upwards with a velocity 20 m/s. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? ($g = 10$ m/s^2)

a) 30 J b) 40 J c) 10 J d) 20 J

(d)

Initial velocity u = 20 m/s; m = 1 kg

Kinetic energy = maximum potential energy

Initial kinetic energy = ½ $mu^2 = \frac{1}{2} \times 1 \times 20^2 = 200$ J

$mgh_{max} = 1 \times 10 \times 18 = 180$ J

$\therefore$ Loss of energy due to air friction =200 – 180 = 20 J

27. In which of the following cases the work done increases the potential energy?

(a) Both conservative and non-conservative forces

(b) Conservative force only

(c) Non-conservative force only

(d) Neither conservative nor non-conservative forces

(b)

In case of non-conservative force, the work done is dissipated as heat, sound, etc., i.e., it does not increase the potential energy. But in case of conservative forces, work done is responsible for increasing the potential energy.

28. A bullet having a speed of 100 m/s crashes through a plank of wood. After passing through a plank, its speed is 80 m/s. Another bullet of the same mass and size, but travelling at 80 m/s, is fired at the plank. The speed of the second bullet after travelling through the plank is:

(a) $10\sqrt{7}$ m/s (b) $20\sqrt{7}$ m/s (c) $30\sqrt{7}$ m/s (d) $20\sqrt{5}$ m/s

(b)

Let F be the resistance force offered by the plank of width x.

Using work energy theorem, Fs = ½ $m[v^2 - u^2]$

In the case of first bullet: $Fx = \frac{1}{2} m [(100)^2 - (80)^2]$

Or $Fx = \frac{1}{2} m \times 180 \times 20$ ……… (i)

In case of second bullet: Let V be the final velocity of bullet; then,

$Fx = \frac{1}{2} m [(80)^2 - V^2]$ ……… (ii)

From eqn. (i) and eqn. (ii), we have;

$\frac{1}{2}$m × 180 × 20 = $\frac{1}{2}$m [$(80)^2 - V^2$]

∴ $V^2 = 6400 - 3600 = 2800$

∴ $V = \sqrt{7 \times 400} = 20\sqrt{7}$ m/s

29. Which of the following is true for any collision?
(a) Both linear momentum and kinetic energy are conserved.
(b) Neither linear momentum nor kinetic energy may be conserved.
(c) Linear momentum is always conserved however, kinetic energy may or may not be conserved.
(d) Kinetic energy is always conserved, but linear momentum may or may not be conserved.

(c)

Linear momentum is conversed in all type of collisions but kinetic energy is not conserved in all type of collisions. Kinetic energy is conserved in elastic collisions but not conserved in inelastic collisions.

30. A sphere A of mass 'm' moving with certain velocity hits another stationary sphere B of different mass. If the ratio of velocities of the spheres after collision is $\frac{V_A}{V_B} = \frac{1-e}{1+e}$, where $'e'$ is coefficient of restitution. The initial velocity of the sphere A with which it strikes is

(a) $V_A + V_B$ (b) $V_A - V_B$ (c) $V_B - V_A$ (d) $\frac{(V_B - V_A)}{2}$

(a)

$\frac{v_A}{v_B} = \frac{1-e}{1+e} \Rightarrow e = \frac{v_B - v_A}{v_A + v_B}$ ..... (1)

We have, $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_B - v_A}{u_A - 0}$ .... (1)

From (1) and (b), $u_A = V_A + V_B$.

31. A force of 10 N is applied on a body for 3 s and the corresponding displacement is 6 m. The power of the source is

(a) 20 W (b) 25 W (c) 40 W (d) 50 W

(a)

Velocity, v = $\frac{\text{displacement}}{\text{time}} = \frac{6}{3}$ = 2 m/s

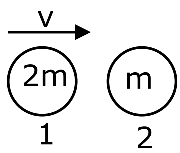
∴ Power P = Fv = 10 × 2 = 20 W.

32. Two billiard balls undergo a head on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed 'v' towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occurred?
(a) Inelastic
(b) Elastic
(c) Completely inelastic
(d) Cannot be determined from the information given

(b)

Let mass of ball 2 be 'm' and mass of ball 1 be 2m.

$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2} \Rightarrow$ e = 1,

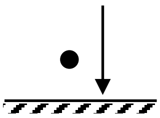


So, collision is elastic

33. A ball is dropped from height 5m. The time after which ball stops rebounding if coefficient of restitution between ball and ground e = ½ is (g = $10 m/s^2$)

(a) 1s (b) 2s (c) 3s (d) infinite

(c)



$v_1 = \sqrt{2gh} = \sqrt{2\times10\times5} = 10\, m/s$.

$\therefore\ t_1 = \frac{v_1}{g} = \frac{10}{10} = 1s$ and also $e = \frac{v_2}{v_1}$

$t_2 = \frac{2v_2}{g} = \frac{2ev_1}{g}$; $t_3 = \frac{2v_3}{g} = \frac{2ev_2}{g} = \frac{2e^2v_1}{g}$

$t = 1 + \frac{2\times e\times10}{10} + \frac{2\times e^2\times10}{10} + ...\ = 1+2[e+e^2+....]$

$t = 1 + \frac{2e}{1-e} = 3s$.

34. A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is

(a) 40 J (b) 60 J (c) 100 J (d) 140 J

(b)

u_1 = 36 km/h = 10 m/s.

Loss in K.E = $\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1 - u_2)^2 = \frac{1}{2}\left(\frac{2\times3}{2+3}\right)(10-0)^2 = 60J$

35. A body of mass 2 kg moving with a velocity of 6 m/s strikes inelastically another body of same mass at rest. The amount of heat evolved during collision is

(a) 36 J (b) 18 J (c) 9 J (d) 3 J

(b)

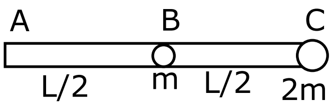
$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$

$(2 \times 6) + (2 \times 0) = (2 + 2)\, v$

Or v = 3 m/s

$K = \frac{1}{2}(m_1 + m_2)\, v^2 = \frac{1}{2} \times 4 \times 9 = 18$ J

36. A long rod ABC of mass 'm' and length 'L' has two particles of masses 'm' and '2m' attached to it as shown in the figure. The system is initially in the horizontally position. The work to be done to keep it vertical with A hinged at the bottom is



(a) 2mgL (b) 3mgL/2

(c) 5mgL/2 (d) 3mgL

(d)

$W = mg\frac{L}{2} + mg\frac{L}{2} + 2mgL = 3mgL$

37. The PE of a certain spring when stretched from natural length through a distance 0.3m is 10J. The amount of work in joule that must be on this spring to stretch it through an additional distance 0.15m will be

(a) 10 J (b) 20 J (c) 7.5 J (d) 12.5 J

(d)

U = ½ kx^2

$\frac{1}{2}k(0.3)^2 = 10 \Rightarrow k = \frac{20}{0.09} = \frac{2000}{9}$

Work done = ½ k $(x_2^2 - x_1^2) = \frac{1}{2}\cdot\frac{2000}{9}[(0.45)^2 - (0.3)^2] = 12.5\, J$

38. A particle is released from rest at origin. It moves under the influence of potential field $U = x^2 - 3x$. The kinetic energy at $x = 2$ is

(a) 2 J (b) 1 J (c) 1.5 J (d) 0 J

(a)

$U = x^2 - 3x$, x = 0, x = 2,

$(U_i)_{x=0} = 0$, $(U_f)_{x=2} = 4 - 6 = -2$

$\Delta k = -\Delta U = 2\,J$.

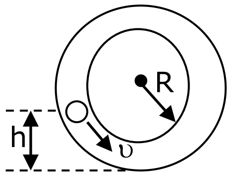
39. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube? Radius of the tube is R

(a) $\sqrt{2g(h+2R)}$

(b) $\frac{5}{2}R$

(c) $\sqrt{9(5R-2h)}$

(d) $\sqrt{2g(2R-h)}$



(d)

Velocity of ball to just reach the top of the tube is given by $0 = v^2 - 2gh_0$

Here, $h_0 = (2R - h)$ and velocity will be zero at topmost point.

Thus, v = $\sqrt{2g(2R-h)}$.

40. A pump ejects 12,000 kg of water at a speed of 4 m/s in 40s. Find the average rate at which the pump is working:

(a) 0.24 kW (b) 2.4 kW (c) 2.4 kW (d) 24 W

(c)

Given m = 12000 kg, v = 4 m/s and t = 40 s

$$P_{avg} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2}\times 12000 \times 4^2}{40} = 2400W = 2.4kW.$$

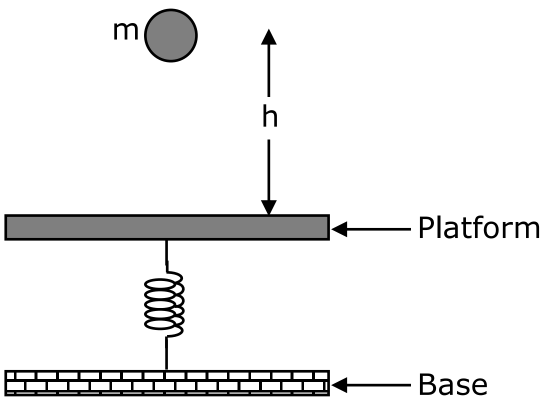
41. A body of mass m = 1kg is dropped from a height h = 40 cm on a horizontal platform fixed to one end of an elastic spring, the other being fixed to a base, as shown in figure. As a result the spring is compressed by an amount x = 10 cm. What is the force constant of the spring. Take $g = 10\,ms^{-2}$

(a) $600\,Nm^{-1}$

(b) $800\,Nm^{-1}$

(c) $1000\,Nm^{-1}$

(d) $1200\,Nm^{-1}$



(c)

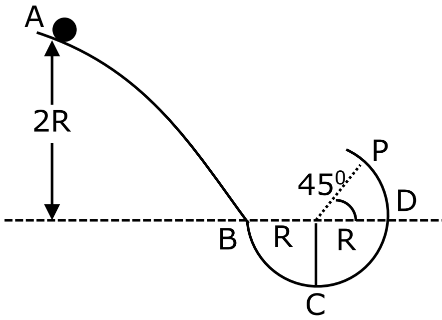
Since the platform is depressed by an amount x, the total work done on the spring is mg (h + x). This work is stored in the spring in the form of potential energy $\frac{1}{2}kx^2$.

Equating the two, we have $\frac{1}{2}kx^2 = mg(h+x)$ or $k = \frac{2mg(h+x)}{x^2}$

Given, h = 0.4m, x = 0.1 m, m = 1kg and $g = 10\,ms^{-2}$.

Substituting these values, we get $k = 1000\,Nm^{-1}$.

42. A particle of mass 'm' slides on a frictionless surface ABCD, starting from rest as shown in figure. The part BCD is a circular arc. If the losses contact at point P, the maximum height attained by the particle from point C is

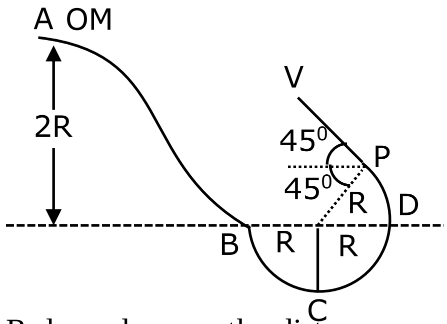


(a) $R\left[2+\frac{1}{2\sqrt{2}}\right]$ (b) $R\left[1+\frac{1}{2\sqrt{2}}\right]R$

(c) 3R (d) none of these

(a)

Applying conservation of energy, we get



$2mgR = mg\frac{R}{\sqrt{2}} + \frac{1}{2}mu^2 \Rightarrow u = \sqrt{4gR - \sqrt{2}gR}$

$H_{\max} = R + \frac{R}{\sqrt{2}} + \frac{u^2 \sin^2 45^0}{2g} = R + \frac{R}{\sqrt{2}} + \frac{4gR - \sqrt{2}gR}{2g \times 2}$

$= R\left[2 + \frac{1}{2\sqrt{2}}\right]$

43. The kinetic energy K of a particle moving along a circle of radius R depends upon the distance s as $K = as^2$. The force acting on the particle is

(a) $2a\frac{s^2}{R}$ (b) $2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$ (c) $2as$ (d) $2a$

(b)

Given that $K = as^2$ or $\frac{1}{2}mv^2 = as^2$ or $mv^2 = 2as^2$

Differentiating w.r.t time, we get $2mv \times \frac{dv}{dt} = 2a \times 2s \times \frac{ds}{dt}$

But $\frac{ds}{dt} = v$

So $2m\frac{dv}{dt} = 4as$ or $m\frac{dv}{dt} = 2as$

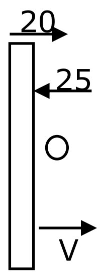
Now, $m\frac{dv}{dt}$ = tangential force = F_t

$F_t = 2as$

Centripetal force = $F_r = \frac{mv^2}{R} = \frac{2as^2}{R}$.

$F_{net} = \sqrt{F_t^2 + F_r^2} = \sqrt{(2as)^2 + \left(\frac{2as^2}{R}\right)^2} = 2as\sqrt{1+\frac{s^2}{R^2}}$.

44. A truck moving on horizontal road east with velocity $20\,ms^{-1}$ collides elastically with a light ball moving with velocity $25\,ms^{-1}$ along west. The velocity of the ball just after collision is



(a) $65\,ms^{-1}$ towards east

(b) $25\,ms^{-1}$ towards west

(c) $65\,ms^{-1}$ towards west

(d) $20\,ms^{-1}$ towards east.

(a)
For elastic collision $u_1 - u_2 = v_2 - v_1$
$20 + 25 = v_2 - 20$
$v_2 = 65$ m/s towards east

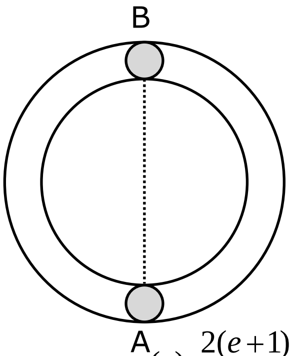
45. A freely falling body takes 4s to reach the ground. One second after the release, the percentage of its potential energy that it still retained is
(a) 6.25% (b) 25% (c) 37.5% (d) 93.75%
(d)

Height of fall, $H_1 = \frac{1}{2}gt^2 = \frac{1}{2}g \times 4^2 = 8g$

After one second, body is at a height of $H_2 = 8g - \frac{1}{2}g \times 1^2 = \frac{15g}{2}$.

% of P.E. retained = $\frac{H_2}{H_1} \times 100\% = \frac{15}{16} \times 100 = 93.75\%$

46. Two equal sphere A and B lies on a smooth horizontal circular groove at opposite ends of a diameter. At time $t = 0$, A is projected along the groove and it first impinges on B at time $t = T_1$ and again at time $t = T_2$. If e is the coefficient of restitution, the ratio T_2 / T_1 is

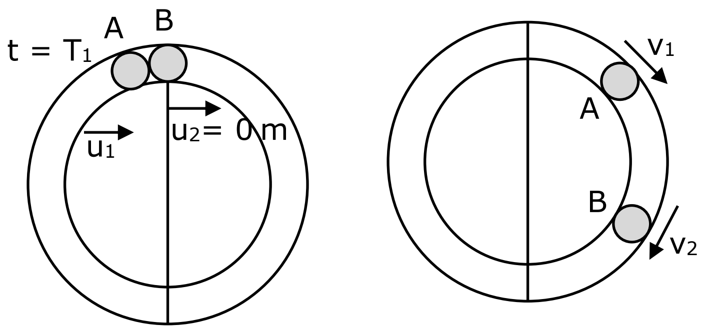


(a) $\frac{2}{e}$ (b) $\frac{(2+e)}{2}$ (c) $\frac{2(e+1)}{e}$ (d) $\frac{(2+e)}{e}$

(d)

$T_1 = \frac{\pi R}{u_1}$ …. (1)

$\frac{v_2 - v_1}{u_1} = e \Rightarrow v_2 - v_1 = eu_1$



Time taken to collide A and B again is

$T_2 - T_1 = \frac{2\pi R}{v_2 - v_1} \Rightarrow T_2 - T_1 = \frac{2\pi R}{eu_1}$ ….. (2)

Dividing (2) by (1), we get $\frac{T_2}{T_1} = \frac{2+e}{e}$.

47. The potential energy (in joule) of a body of mass 2kg moving in the xy-plane is given by $U = 6x + 8y$ where the position coordinates 'x' and 'y' are measured in metre. If the body is at rest at point (6m, 4m) at time t = 0, it will cross the y-axis at time 't' equal to

(a) 1 s (b) 2 s (c) 3 s (d) 4 s

(b)

Given U = 6x + 8y joule and mass m = 2kg. Force along x-axis is

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}(6x+8y) = -6\,N$$

Force along y-axis is $F_y = -\frac{dU}{dy} = -\frac{d}{dy}(6x+8y) = -8N$

Therefore, the 'x' and 'y' components of acceleration are

$a_x = \frac{F_x}{m} = \frac{-6}{2} = -3ms^{-2}$ and $a_y = \frac{F_y}{m} = \frac{-8}{2} = -4ms^{-2}$

Resultant acceleration $a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-3)^2 + (-4)^2} = 5ms^{-2}$

The 'x' and 'y' coordinates of the body at time 't' are

$x = x_0 + \frac{1}{2}a_x t^2 = 6 - \frac{1}{2}\times 3\times t^2 = \left(6 - \frac{3}{2}t^2\right)$ metre

and $y = y_0 + \frac{1}{2}a_y t^2 = 4 - \frac{1}{2}\times 4\times t^2 = (4 - 2t^2)$ metre.

The body will cross the y-axis when x = 0, i.e., at time 't' given by

$\left(6 - \frac{3}{2}t^2\right) = 0$ or t = 2s.

48. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain 'n' times water from the same pipe in the same time, by what amount should the power of the motor be increased?

(a) n^2 times (b) n^3 times (c) n times (d) $n^{3/2}$ times

(b)

If a liquid of density ρ is flowing through a pipe of cross section A at speed *v*, the mass coming out per second will be $\left(\frac{dm}{dt}\right) = \rho Av$

In order to get 'n' times water in the same time, we get $\left(\frac{dm}{dt}\right)' = n\left(\frac{dm}{dt}\right)$

i.e., $A'V'\rho = nAV\rho$

But as pipe and liquid are same, $\rho' = \rho$, $A' = A$, $V' = nV$.

So, $\frac{F'}{F} = \frac{V'\left(\frac{dm}{dt}\right)'}{V\left(\frac{dm}{dt}\right)} = \frac{nV\frac{ndm}{dt}}{V\left(\frac{dm}{dt}\right)} = n^2$

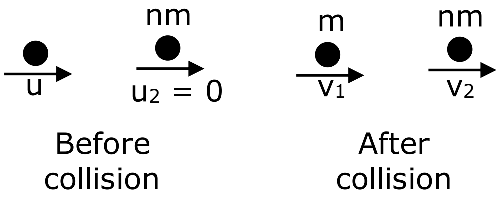
$\frac{P'}{P} = \frac{F'V'}{FV} = \frac{(n^2F)(nV)}{FV}$ or $P' = n^3P$.

49. A ball of mass m moving with speed u undergoes a head-on elastic collision with a ball of mass nm initially at rest. The fraction of the incident energy transferred to the second ball is

(a) $\frac{n}{1+n}$ (b) $\frac{n}{(1+n)^2}$ (c) $\frac{2n}{(1+n)^2}$ (d) $\frac{4n}{(1+n)^2}$

(d)

As the collision is elastic, we can find $v_1 = \frac{1-n}{1+n}u$, $v_2 = \frac{2u}{1+n}$ using velocities after collision formula



Hence, the required fraction is $\frac{\frac{1}{2}nmv_2^2}{\frac{1}{2}mu^2} = n\left(\frac{v_2}{u}\right)^2 = \frac{4n}{(1+n)^2}$

50. Which of the following does not hold when two particle of masses m_1 and m_2 undergo elastic collision?

(a) When $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of kinetic energy in head on collision.

(b) When $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of momentum in head on collision.

(c) When $m_1 >> m_2$ and m_2 is stationary, after head on collision m_2 moves with twice the velocity of m_1.

(d) When the collision is oblique and $m_1 = m_2$ with m_2 stationary, after the collision the particle moves in opposite directions.

(d)